



[The Economics of Search Engines – Search, Ad Auctions & Game Theory](#)

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[www.issuu.com/jacobhansen/docs/theeconomicsofsearchengines](http://www.issuu.com/jacobhansen/docs/theeconomicsofsearchengines)

*“It is clear that a search engine which was taking money for showing cellular phone ads would have difficulty justifying the page that our system returned to its paying advertisers[...]. We expect that advertising funded search engines will be inherently biased towards the advertisers and away from the needs of the customer”*

Larry Page and Sergey Brin - The Founders of Google - in their seminal paper on PageRank in 2000

## **Executive Summary**

This project examines the economics of search engines. It is found that the business model of search engines is based on advertising revenues. This is called search engine marketing. These advertising revenues are generated through an auction where advertisers bid on keywords, a mechanism known as an ad auction. The main focus of this project is an investigation of the challenges that exist in the context of ad auctions. One major challenge that advertisers face is how much to bid for keywords in ad auctions. Therefore we present a practical bidding strategy to advertisers to address this problem. The conclusion is that advertisers should follow a marginal bidding strategy.

We start out by investigating the business of search engines. It is conveyed that search engines act as an intermediary between users, advertisers and content providers. We argue that search engines are a scale-intensive business with high fixed costs and low marginal costs.

The fundamentals of search engine marketing are presented next. We find that a search engine generates revenue by selling ads that are priced on a cost-per-click basis, which means that the advertiser has to pay each time a user clicks on their advertisement. We study how the ad placement on a search engine result page is a key factor in determining how many clicks an ad receives. The position receiving most clicks is the first position, followed by the second and third position. Due to this, higher positions on a search engine result page are more valuable to advertisers.

This project outlines the theoretical foundation of auction and game theory and present why auctions are useful as a pricing mechanism of ads. We conclude that ad auctions today are conducted as a second-price-auction where bidding takes place continuously. Furthermore we introduce a game-theoretic model of ad auctions titled position auctions. We investigate ad auctions as a game and conclude that the current format of ad auctions does not have equilibrium in dominant strategies. Furthermore, the model of position auctions shows that advertisers contemplating entering into an ad auction face a 'supply curve of clicks'. This means that the better the placement of an ad on a search engine result page is, the higher the price-per-click is in that position. In order for advertisers to maximize profit when bidding in ad auctions they must take marginal cost-per-click into consideration. This can be achieved by following the marginal bidding strategy presented in this project.

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# 1. Introduction

For 80%<sup>1</sup> of all internet users the first stop on the web is a search engine. Some industry experts predict that search engines will become the new interface of commerce. Prior to purchasing a product, a consumer will actively search for it; consumers search the web to find cheap flight tickets and search their GPS for a nearby gas station.

*“We think as shopping as basically an application of search” - Yahoo!*

There are three search engines among the top five most visited web sites: Google (1), Yahoo (2) and Bing (5)<sup>2</sup>. Search engines have become gatekeepers to businesses on the internet. Moreover, search engine marketing has become a billion dollar industry. The total revenues of search giants like Google and Yahoo in 2008 totaled approximately \$30 Billion<sup>3</sup>. Additionally, the combined market capitalization of the search giants Google and Yahoo is approximately \$160 Billion which corresponds to the total nominal GDP of countries such as the Philippines or Pakistan. These facts are the main reasons to give search engines and online advertising some attention.

Today the most popular method of advertising online is displaying an ad on a search engine with relevance to the keywords a user types in a search box. For example if someone types in the keyword “laptop” in a search box, an ad for laptops will appear next to the search results. When a user clicks on the ad, he is directed from a search engine to a website via a link. This is called search engine marketing, and is a technique for advertisers to generate traffic from search engines to websites. These advertisements selling billion dollars every year represent more than half of all internet advertising expenditures. Presently Google, Yahoo and Microsoft are the market leaders in the search industry, where they have been able to generate profits through advertisements.

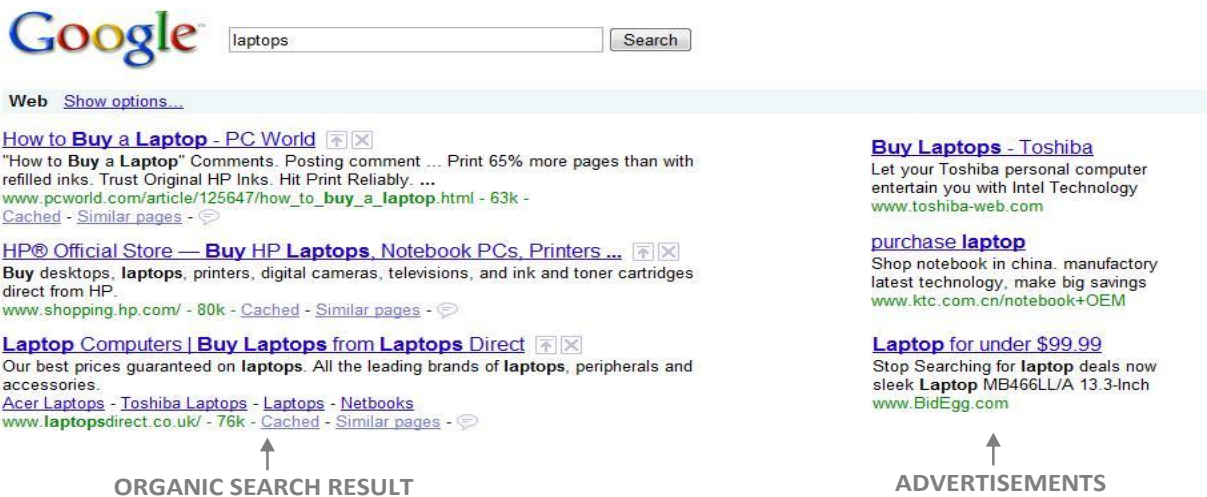
The pricing structure is such that every time a user clicks on an ad, the advertiser has to pay the search engines a small amount. This price is not fixed and fluctuates from minute to minute from keyword to keyword. Auctions which we are familiar with from sale of antique paintings or used bikes, turn out to be useful to sell advertisements online. For that reason it has become the industry standard today to sell ads on search engines through an auction. These auctions are referred to as ad auctions.

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<sup>1</sup> (Battelle, 2005)

<sup>2</sup> (Alexa web statistics, 2009)

<sup>3</sup> (See appendix)



**FIGURE 1** - Google Search Result Page

The ad auction system and the huge investments in search engine marketing has led to that the Google Ad Auction has being coined: “The world biggest auction”. The Google advertising system processes hundreds of millions of auctions per day, because every search query Google processes, involves the automated execution of an auction. For the most competitive keywords advertisers are paying as much as \$50 for each click their ad receives<sup>4</sup>.

Historically advertisement budgets have not been subject to much scrutiny. Nevertheless digitization and the rise of the internet have led to easy data gathering, and advertisement budgets are today increasingly exposed to measurements and accountability. This enormous wealth of data, in addition to the rise of Google and Yahoo has drawn statisticians and economists into the field of online marketing. Terms such as cash flows analysis, ROI, profitability, margins, auctions and bids are common within the field. Marketing has become data-driven.

This project is a part of this trend. We are all familiar with Google, Yahoo and Bing – but how about the underlying economics? Not only is sponsored search the main revenue source for search engines, but it is also important to businesses of all types and sizes. Therefore, several questions arise; Are ads sold in the most appropriate manner? How should advertisers manage in this field? Which experiences can we collect from ad auctions?

To succeed in this field, advertisers need to understand the economics of search engines and how ads are sold through ad auctions. This project seeks to investigate search engines, search engine marketing and ad auctions, using concepts and methods from the field of finance and economics.

<sup>4</sup> (Spyfu, 2009)



## 2. Problem Statement

This project is an exploration into the economics of search engines. The primary goal is an investigation of ad auctions, which challenges arise in relation to ad auctions and how advertisers can manage these challenges. From this, following problem statement occurs:

Which challenges exist in the context of ad auctions and how can advertisers respond to these challenges?

To answer the above question I find it necessary to address the following sub questions:

- What is the business model of search engines?
- How are ad auctions structured?
- Which challenges arise in relation to ad auctions?
- What recommendations can be communicated to advertisers?

### 2.1 Purpose of the project and target group

The project targets readers familiar with the user-side of search engines but unfamiliar with the business model and ad auctions. It is expected that the reader is familiar with basic microeconomic theory such as marginal cost functions, the pursuit of profit maximization and utility concepts.

This project marks the end of my Master Program at the Copenhagen Business School in Applied Economics and Finance.

### 2.2 Delimitation

One can apply a technical, creative or business perspective when studying search engines. While these perspectives certainly overlap, it is difficult to view them exclusively. However, the focus of this project is to study the business perspective of search engines. Therefore, technical elements, such as search engine optimization (SEO), and creative issues such as the layout and quality of ads will not be addressed in this project. However to understand search engines and ad auctions in detail it is encouraged that the reader do further research on these topics.

Furthermore this project primarily addresses the relationship between the search engine and the advertiser. Though the user plays an important role in this market we will not treat the user explicitly. Lastly, in the end of July 2009 Microsoft announced a partner deal with Yahoo. This will not be treated in this project, and Microsoft and Yahoo! are treated as two different entities.

### 2.3 Structure of the project

This project is structured in three parts. The first part includes sections 4-9 and sets the background by introducing the problem of ad auctions. In sections 4 and 5 we lay out the fundamentals of search engines and investigate the business of search engines. Sections 6 and 7 explain the media model and the evolution of search engine marketing. Finally, the first part ends with sections 8 and 9 which present the fundamentals of search engine marketing and ad auctions.

The second part consists of sections 10-17 and introduces and discusses a model of ad auctions. Sections 10 and 11 sketch out the theoretical foundation of auction and game theory and present why auctions are useful as a pricing mechanism. Section 12 investigates the environment of ad auctions. Sections 13 and 14 examine ad auctions as a game and establish the foundation of the model of ad auction which is presented in section 15. In section 16 we discuss the pros and cons of the underlying assumptions of position auctions and finally we end part two with section 17 and a discussion of the challenges that search engines face when designing ad auctions.

The third part includes section 18-21. Based on our previous findings, section 18 presents a practical bidding strategy for advertisers to utilize and section 19 concerns various practical issues regarding search engine marketing. Lastly, section 20 summarizes the overall conclusions. Part three ends with section 21 with an outlook into the future of search engines and ad auctions.

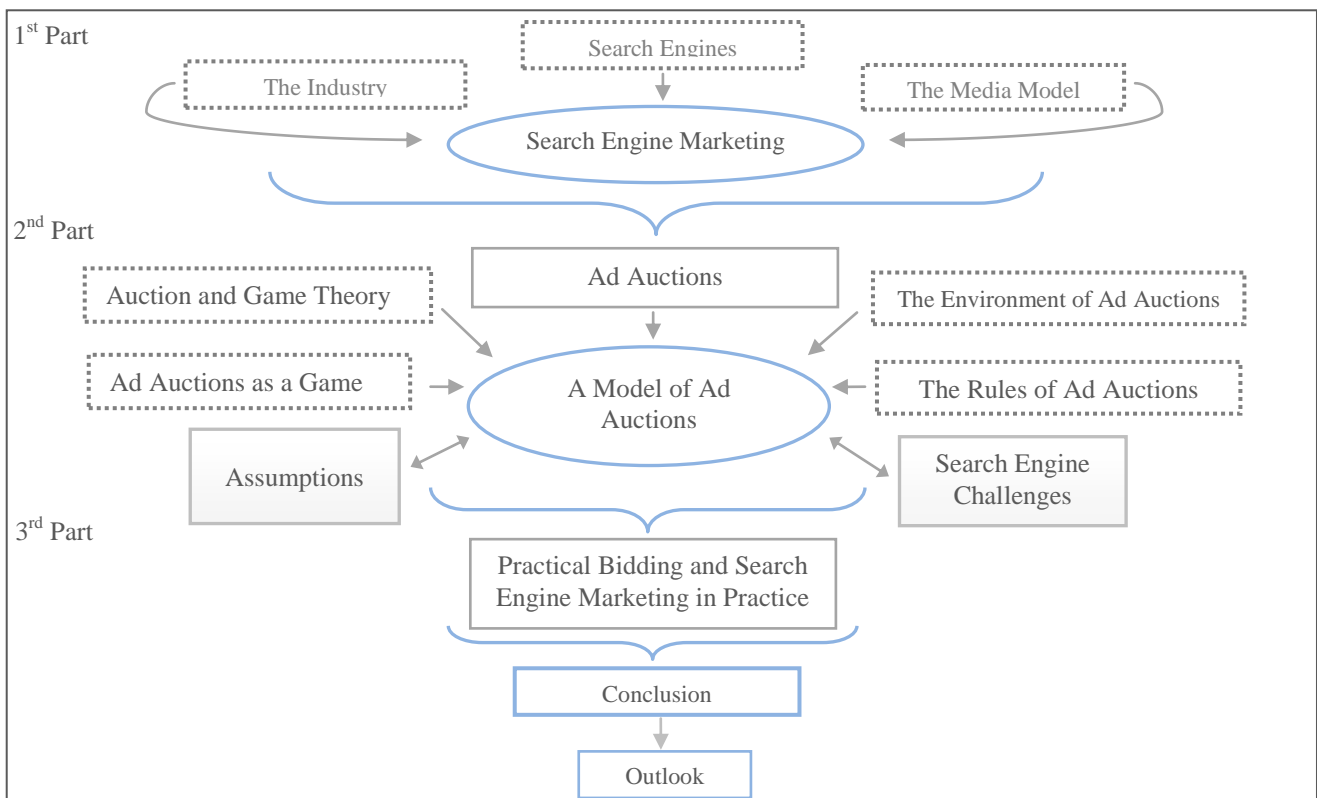


FIGURE 2 – Structure of the project

### 3. Methodology

This project is a theoretical exploration of ad auctions examined in the context of search engine marketing. Though this is primarily a theoretical examination of ad auctions there is an emphasis on how ad auctions have emerged, how they are applied today and the practical challenges that advertisers are facing.

Two papers have had a great importance on this project. The first is by Hal Varian (2006a) on Position Auctions. Hal Varian is from Berkeley and recognized within the field of information economics. He is currently Chief Economist at Google. The second paper is by Benjamin Edelman, Michael Ostrovsky and Michael Schwarz (2005) on the Generalized Second Price Auction. Benjamin Edelman is a Harvard professor and Michael Ostrovsky is from Stanford. Lastly Michael Schwarz is a Berkeley professor and a principal research scientist at Yahoo!. They have all done numerous publications within the field of ad auctions. The two papers are closely related and have been the theoretic fundament for my model of ad auctions. Furthermore the two papers are widely cited within the ad auction literature and numerous have expanded and added to their work. I consider these papers to be reliably.

In order to understand ad auctions in detail I present the fundamental reasoning and terminology of game and auction theory in section 10. My primary source on auction and game theory has been Bierman and Fernandez's (1998) book on game theory.

During the project I have been in contact with Google as well as the Danish search engine Jubii. Unfortunately it has not been possible to access data through these channels. Therefore I have not been able to support the theories by primary data. However both Hal Varian (2006a) and Edelman et. al. (2005) have both done empiric studies which I have represented when relevant.

Although I have not been able to collect primary data I have had the fantastic opportunity to gather firsthand experience with search engine marketing. This has been achieved during the last three months of the project, where I, as part of my job, managed several search engine marketing campaigns. This means that I have been an active player (bidder) in the auction mechanism and I experienced the practical side of ad auctions. Based on my experiences this paper ends with a discussion of some of the challenges that search engine marketers' face.

One interesting aspect of search engines and ad auctions is that the literature is evolving almost on a daily basis. For example during the course of the project, in March 2009, the first Wikipedia article appeared on Generalized Second Price Auctions which correspond to the position auctions examined in this paper. Also in June 2009 was Bing, a new search engine by Microsoft, released. In order to be up-to-date with this development I have during the project been an active reader on the most influential blogs within the industry such as searchengineland.com and battellemedia.com.

This project is available online at:

[www.issuu.com/jacobhansen/docs/theeconomicsofsearchengines](http://www.issuu.com/jacobhansen/docs/theeconomicsofsearchengines)

# Part I

The economics of search engines and search engine marketing are introduced in this part. In addition ad auctions are introduced and placed in the context of search engine marketing. This part I, is primarily targeted the reader unfamiliar with online advertising, search engine marketing and ad auctions and serves as a foundation of part II and part III.

## 4. Understanding Search Engines

This section disentangles the different components of search engines and why they are valuable.

### 4.1 Search Engines – Sorting Information

The information available to consumers on the internet is overwhelming. One can distinguish between the surface web (which is indexed by search engines) and the deep web (which is not indexed by search engines). In May 2009, the indexed web contained around 25.36 billion web pages<sup>5</sup>. To this one can add the deep web which contains around 500 times more web pages than the indexed surface web. This vast amount of information is simply too much for consumers to grasp without assisting tools. According to Alexa web statistics, the top five sites on the internet in July 2009 were:

1. Google.com
2. Yahoo.com
3. Youtube.com
4. Facebook.com
5. Bing.com

Among the top five sites are seen three search engines (Google, Yahoo and Bing). We see that search engines are the tool users employ to sort the information available on the web. Search engines therefore become navigational tools for users to navigate the internet.

*“A wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it.”*

Herbert Simon, 1971.

In an economy with millions of books, millions of songs, millions of films, millions of applications, millions of everything that request our attention - being found is valuable. Due to this enormous amount of information search engines have become the gatekeepers for online businesses as the entry point for consumers contemplating an economic transaction. Attention economics, which treat human attention as a scarce commodity, have had an increasing presence in recent economic literature. One could argue that search engines are in the business of allocating consumer's attention. Applegate et al. (2007) describes how early internet portals emerged to help consumers gain access to the increasing amount of content available on the web. Portals, and recently search engines, provide the tools necessary for users to find direction on the internet. With technology,

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<sup>5</sup> (worldwidewebsize.com, 2009)

search engines have been able to sort information and help users find what they are looking for. Secondly, search engines have become valuable because they are able to drag consumers' attention into the arms of businesses. Search engines are therefore treating consumer attention as a property of the search engines, thus an asset which they can sell.

#### **4.2 Search is Information about User Intentions**

Consumers use search engines as means to an end. When a user types a query in a search engine he is revealing information about his intentions. If multiple users search for "yellow laptops" there might be a demand for yellow laptops. The aggregated queries that users type in a search engine provide knowledge about consumers needs, because the queries represent what the consumers are demanding. This information is valuable to businesses. Battelle (2005) argues that this intentional traffic from consumers, the queries and click stream, is an asset to search engines. The aggregated information about consumer intentions is an insight into the mind of consumers. There are several tools available for advertisers to analyze and understand users' search queries. One example is Zeitgeist (German: the spirit of the time) which is a service provided by Google that analyzes search trends and can help advertisers with information about what is "in" and what is "hot".

#### **4.3 Search Engines – How?**

A search engine consists of three major elements (see **FIGURE 4** on the next page):

- The Crawl
- The Index
- The Runtime System (query processor)

The crawls are automated programs often called "bots" or "spiders" that use hyperlinks between web pages to "crawl" and index web pages. From this information the crawl create a database of web pages which is referred to as the index. The runtime system is the interface and software that connects a user's query to the index. The runtime also manages the question of relevance and ranking. There are several approaches to organize search, but currently each major search engine is organized in this manner. The concept of using links to organize and evaluate web pages relevance has been drawn from the academic world. Academic publishing depends on evaluation by peers to determine academic importance and relevance (peer review). The entire web is loosely based on the premise of citation and annotation. You can think of a link as a citation and the text describing the link as an annotation.

#### 4.4 Organic search and advertisements

Search engines organize information on the internet. The search result based on the query of a user is known as the organic search result. The organic search result has attracted the attention of consumers. Consumers' attention is valuable to advertisers who are willing to pay for displaying ads on search engines.

Therefore when a user visits a search engine two different sections are displayed: The organic search result, and ads related to the search query. This is the business model of all major search engines.

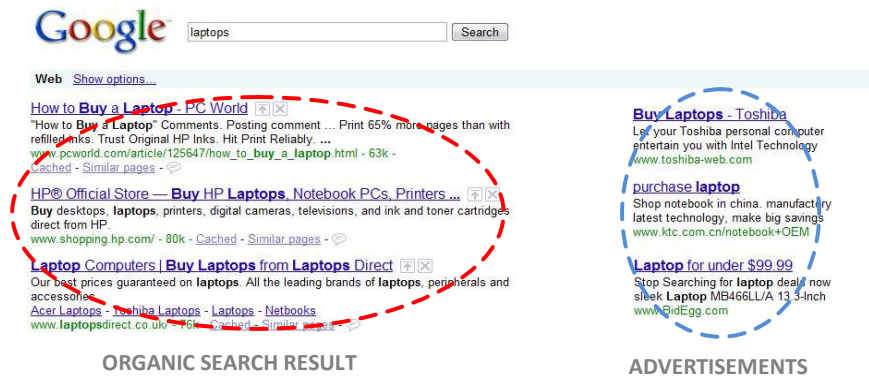


FIGURE 3 – Organic search and advertisements

#### 4.5 Search Engines – Intermediaries

Search engines are acting as an intermediary between users, content providers and advertisers. Due to the enormous amount of information on the internet, there is a need from users to sort and extract relevant information. This is the service that search engines are providing. Search engines are therefore in the service business because they are delivering an information product. According to Shapiro & Varian (1999) this is also an experience good because the users have to experience the search result, before they can value it. Users' attention is an asset and search engines are selling this asset to advertisers, who are competing for the users' attention. Search engines are therefore operating in a matching market where they match the attention of users with advertisers.

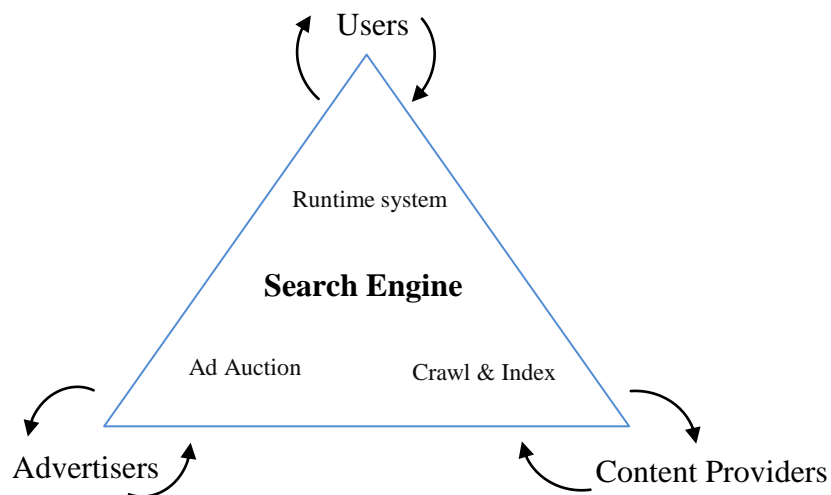


FIGURE 4 – Search Engine as an Intermediary



## 5. The Industry of Search Engines

Generally search engines can be divided into two categories: horizontal search engines and vertical search engines. A horizontal search engine has a broad market focus whereas a vertical search engine focuses on a niche market like cars or books. On a worldwide scale the three dominant players in the search industry are Google, Yahoo and Bing (Microsoft). Which each respectively have a world-wide market share of 62%, 12% and 3%<sup>6</sup>. However in some local markets they face strong competition from local search engines such as Yandex in Russia or Baidu in China. In addition new search engines are popping up almost on a daily basis to compete with the giants.

Both Google and Yahoo are examples of new businesses built on the internet. Search engines are operating networked business due to the high importance of a network of partners and advertisers which was alluded to earlier. Though Yahoo started out as a portal, and Google as a search engine, both are converging into the same space; they mediate information and services for consumers, and derive value from those services using the traditional revenue streams of the media business – advertising and subscriptions.

Search engines compete on different levels and are a part of a larger competitive landscape. As discussed in section 4 search engines act as intermediaries between users, advertisers and content providers. This means that search engines are competing:

- For access and ownership of user traffic
- For returning users through good organic search result
- For a large network of advertisers
- For access to content

Search engines must have access to user traffic. Access to user traffic can be achieved by having a strong brand as Google or Yahoo or through partnerships with other websites that direct users to the search engine. For example, Google has a partnership deal with AOL regarding search and advertisements. If a user is not satisfied with the search result they will not return to the search engine. For that reason a high-quality organic search result is necessary to have returning users. Search engines also compete for advertisers who can choose to advertise on different search engines. A large network of advertisers is required in order to have a broad portfolio of ads to display on the search engine. Lastly, search engines also face competition from content providers (publishers) to whom advertising revenues are a strategic revenue stream, and who is not willing to

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<sup>6</sup> (Comscore, 2007)

let search engines take over this market. None of these different perspectives can be seen in isolation. To attract advertisers, search engines must attract users, and to attract users, they must have good organic search results, and so on. One could argue that the main competitor to Yahoo! sponsored search is Google's organic search results (and vice versa).

Varian (2006b) argues that search engines by nature are a scale intensive business. The reason is that fewer than one out of a thousand users who see an online ad actually buy the product. As a result search engines have to pay large fixed costs to build the scale necessary to serve enough ads to cover the entry costs of serving a large audience. However when a search engine has built the infrastructure to serve a reasonable audience the marginal cost of expanding is small. The marginal cost of performing one extra search and displaying one extra advertisement is negligible. These characteristics can be seen in the revenue and cost structure of the industry. Below you can see the development in revenues for Google, Yahoo and Microsoft search during the last years.

<b>Google</b>	<b>Year Ended December 31</b>				
	<b>2004</b>	<b>2005</b>	<b>2006</b>	<b>2007</b>	<b>2008</b>
			<b>(in millions USD)</b>		
<b>Revenues</b>	<b>3.189</b>	<b>6.139</b>	<b>10.605</b>	<b>16.594</b>	<b>21.796</b>
<b>Total cost and expenses</b>	<b>2.549</b>	<b>4.121</b>	<b>7.055</b>	<b>11.510</b>	<b>15.164</b>
<b>Income from operations</b>	<b>640</b>	<b>2.017</b>	<b>3.550</b>	<b>5.084</b>	<b>6.632</b>
<b>Net income</b>	<b>399</b>	<b>1.465</b>	<b>3.077</b>	<b>4.204</b>	<b>4.227</b>

<b>Yahoo!</b>	<b>Year Ended December 31</b>				
	<b>2004</b>	<b>2005</b>	<b>2006</b>	<b>2007</b>	<b>2008</b>
			<b>(in millions USD)</b>		
<b>Revenues</b>	-	-	<b>6.426</b>	<b>6.969</b>	<b>7.209</b>
<b>Total cost and expenses</b>	-	-	<b>2.809</b>	<b>3.435</b>	<b>4.172</b>
<b>Income from operations</b>	-	-	<b>941</b>	<b>695</b>	<b>13</b>
<b>Net income</b>	-	-	<b>751</b>	<b>660</b>	<b>424</b>

<b>Microsoft Online Business Service</b>	<b>Year Ended December 31</b>				
	<b>2004</b>	<b>2005</b>	<b>2006</b>	<b>2007</b>	<b>2008</b>
			<b>(in millions USD)</b>		
<b>Revenues</b>	-	-	<b>2.296</b>	<b>2.441</b>	<b>3.214</b>
<b>Total cost and expenses</b>	-	-	-	-	-
<b>Income from operations</b>	-	-	<b>5</b>	<b>(617,00)</b>	<b>(1233,00)</b>
<b>Net income</b>	-	-	-	-	-

**TABLE 1:** Summary of Financial Information, Google, Yahoo & Microsoft Online Business Service. (see appendix).

Google's receives 98% of their revenue from advertising<sup>7</sup>, whereas Yahoo state in their annual report that around 90% percent of Yahoo's revenue is from advertising and marketing activities. This pattern is also true for Microsoft Online Business Service, which is a Microsoft unit only focussed on the search business. Search engines' main cost is traffic acquisition costs, which are payments made to affiliates and websites that direct users to the search engine. Often search engines engage in partnerships with large content providers or portals in order to attract traffic to their website. In these cases, the search engines normally split the revenues with their partners. For a more detailed financial overview see the appendix I and II.

It is evident that Google, Yahoo and Microsoft have experienced a significant revenue growth the last several years. Yahoo and Google state in their annual report that this growth is due to more clicks and not due to higher prices. (Note that during August 2009, Yahoo and Microsoft announced a partnership deal which somehow changes the landscape for search engines, see section 2.2).

The cost for users to switch between search engines is minimal. This is true also for advertisers who can easily carry out several campaigns with different search engines. For this reason also Google states in their 2008 annual report that they expect the revenue growth rate to decline and they anticipate a downward pressure on the operating margin due to increased competition.

To conclude, we see that online advertising is a scale intensive business and the characteristics of the search engine industry are:

- Advertiser supported business model
- High fixed costs – due to infrastructure development
- Low marginal costs
- Requirement of a mass market and a brand
- Necessary with access to content
- Low switching costs among users

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<sup>7</sup> (Google, 2008)

## 6. The Media Model

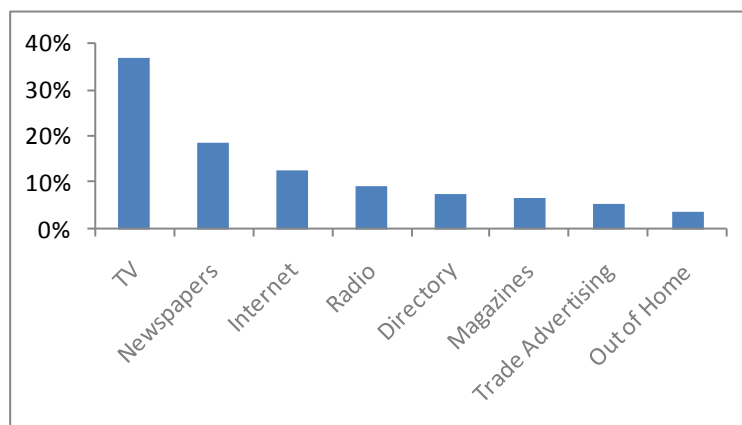
Battelle (2005) argues that for a large part of the IT industry search engines can trace its roots back to the academic labs at universities. Search engines like Google, Yahoo and Excite all emerged from Stanford. Another search engine, Inktomi, which later was acquired by Yahoo! emerged from Berkeley. Search engines are by nature a technical business, and as many other online businesses search engines have been struggling to find a viable business model. It turned out that the business model known as the media model is successful for search engines too.

### 6.1 The Media Model

We have established that the business model of search engines relies on revenue from advertisements. This business model is characterized as the media model and is well-known from other media such as radio, newspapers and TV. The media model started out in the print media, moved on to radio, television and later the internet. The internet is therefore not the first time advertisements have been digitalized but rather a natural step in a continuing evolution. When search engines exercise the media model, advertisements are applied in a perfect symbiosis with search and targeted each consumer individually. Furthermore, collection of data relating consumer activity has made it possible to precisely measure the effectiveness of ad campaigns.

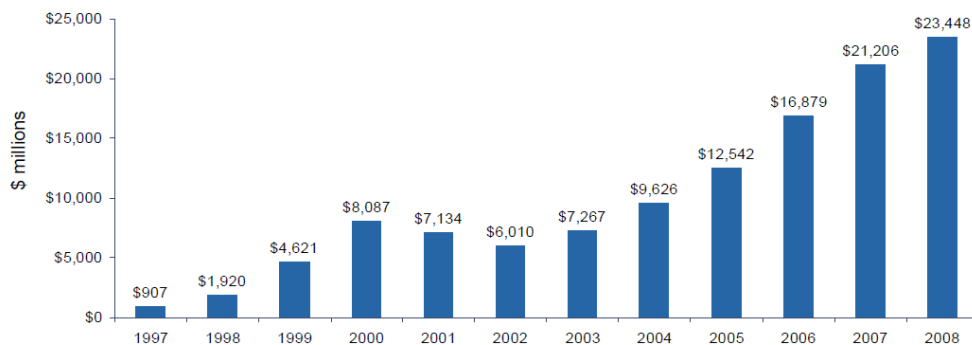
### 6.2 Internet Advertisement

According to PWC (2009) the total US advertising revenue in 2008 accounted to \$187 Billion. Of this, internet advertising expenditure accounted to \$23,4 Billion which corresponds to a market share of 13%. This is shown in the graph below which depicts the percentage of US media expenditures across different media channels in 2008. We see that TV is still the most important advertising channel and that newspapers also play a noteworthy role. However internet advertising has experienced fairly high growth rates during the last several years. The study from PWC (2009) also explains that the rate of growth of internet advertising is significantly higher than the rate of growth when cable television



**FIGURE 5:** U.S. Advertising expenditures across different media 2008, percentage. (PWC, 2009)

advertising was introduced. From the figure below we see that internet advertising almost has doubled from 2005 to 2008.



**FIGURE 6:** Annual \$ internet advertising revenue 1997-2008. (PWC, 2009)

As internet usage continues to grow, the medium is going to take on an increasing share of media budgets. Despite the downturn internet advertising experienced growth in 2008, with advertising expenditures rising by 13%<sup>8</sup>. There is a similar growth forecast for 2009. According to James (2009) internet advertising is thus set to be the main winner of the current downturn.

### 6.3 Pull vs. Push Marketing

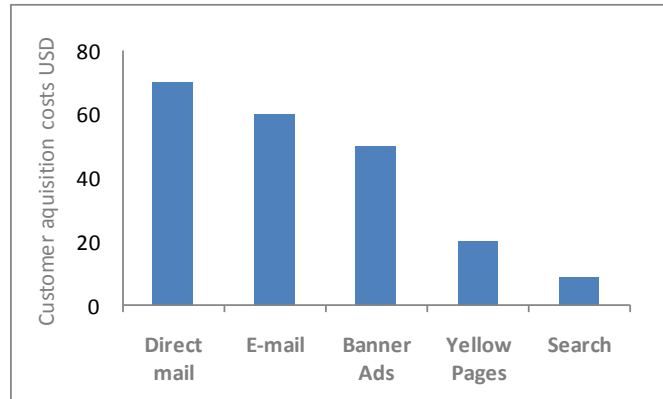
The ads on search engines are directly correlated to the keyword a user types into the search box. This means that if you search for “laptops” the search engine will display an ad for laptops. This keyword advertising has enabled advertisers to target customers already interested in their products. A user who searches for “laptops” has already indicated that he is interested in laptops. This is known as pull marketing. Contrary to pull marketing is push marketing. In push marketing advertisers create awareness of a product by pushing it in front of the user. TV advertisements are one example of push marketing. In TV ads you “push” a message to the consumer, they are not necessarily requesting or interested in.

One should expect that because search engine marketing is pull marketing it should be more effective than push marketing. This is also the case. The customer acquisition cost for search engine marketing is relatively low compared to other marketing channels. Customer acquisition costs measures how much it cost in terms of advertising expenditures to acquire one new customer. For example if you spend \$1000 to get 100 new customers the customer acquisition cost is \$10. The chart on the next page compares customer acquisition cost across different channels. Note that

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<sup>8</sup> (PWC, 2009)

search is the cheapest channel. We also see that the customer acquisition cost for banner ads<sup>9</sup> are higher than for search. The reason is that banners ads are push marketing and they can in some dimensions be compared to traditional TV ads. One problem with banner ads is that advertisers are not able to precisely target consumers. Banner ads are therefore often associated with some noise. This has the implication that advertisers are only willing to pay a smaller amount per click/impression for banner ads compared to search engine ads. The reason is that search engines generate more relevant clicks and impressions which is a key difference between pull and push marketing.



**FIGURE 7:** Approximate Customer Acquisition Cost across various channels. 2005, USD. (Battelle, 2005)

We discussed in section 5 that less than one out of a thousand consumers who see an ad on a search engine actually buys the product. Despite this seemingly low yield, ads on search engines are one of the most effective forms of advertising. TV ads or newspaper ads are significantly less effective because a smaller fraction of those who see the ad purchases the product being advertised. Another difference which Donaldson (2008) points out is that with search engine marketing it is possible to target a narrow segment of the market. When you target a small segment it does not require large marketing investments contrary to for example TV campaigns. This has made it an easy entry point for businesses of all sizes. For example there are more than 100.000<sup>10</sup> advertisers on Google.

Though search engines have emerged as IT tech companies they are converging towards media companies. When looking at Google, it is obvious that their main expertise is the internet advertisement space they. Nonetheless they have also embraced the traditional media. In Google's 2008 annual report they state that following products are available through the Google network:

- Google Audio Ads
- Google Print Ads
- Google TV Ads

Google is therefore expanding from online advertisements into the traditional media space. One example of this is that Google is currently placing ads in more than 650 newspapers in the US.

<sup>9</sup> (see glossary for definition of banner ads)

<sup>10</sup> (Donaldson, 2008)

## 6.4 Advertising Pricing Models

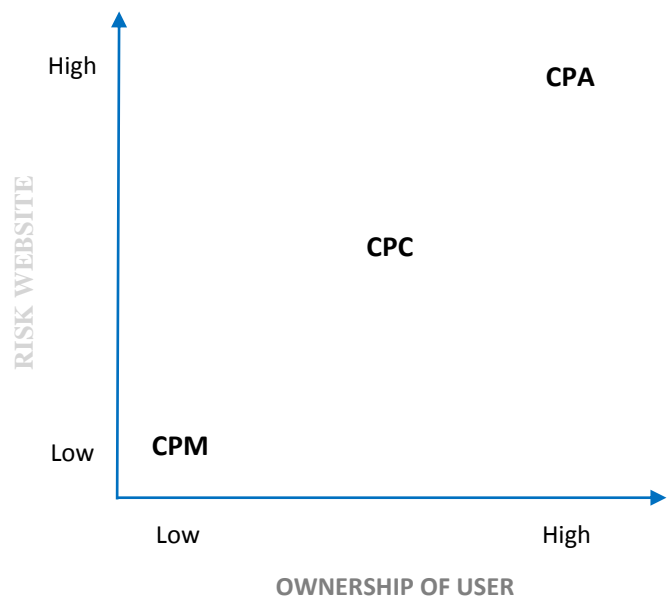
Within internet advertising there exist three different payment models: CPM, CPC and CPA.

**CPM** is an abbreviation of cost-per-mille. This means that for every thousand impressions (views) the advertiser had to pay a predefined amount. The CPM model originates from the print media, and was historically based on how many copies the consumers bought. Traditional print and TV advertisements are priced on a cost-per-mille (CPM) basis. In a CPM model, the advertiser therefore pays every time a user is exposed to his advertisement. In online advertising an impression corresponds to every time a user sees an advertisement on a webpage.

**CPC** is an abbreviation of cost-per-click. In a cost-per-click model, the advertiser is only paying the host website of the ad when the ad is clicked on by a user. This means that if an advertisement gets 1000 impressions but no clicks, the advertiser does not pay anything. The cost-per-click model is by nature an online internet advertising model.

**CPA** is an abbreviation for cost-per-acquisition. In a cost-per-acquisition model, the advertiser is only paying the host website of the ads when he acquires a new customer. This is the ideal model for advertisers because the risk is low.

CPM, CPC and CPA allow for different risk-sharing between the advertiser and the host website of the ad. If we are dealing with the CPM pricing model, the advertiser will have to pay for all users exposed to his ad. In this case the host website bears only minimal risk; because it will receive payment whether or not the user clicks on the advertisement. On the other hand, if the advertiser is paying on a CPA basis the host site is taking ownership of the users' behavior beyond just showing an ad. In this case, the host webpage is bearing the risk by having ownership of the user until an acquisition is executed. Finally the CPC model is a middle ground, the host website and the advertiser share the overall risk of the user.



**FIGURE 8:** The relationship between ownership of user and risk website

## 6.5 Offline and Online Marketing

Several times it has been argued that the amount of data has exploded together with the digitalization of online marketing. This has altered the playing field for online advertisement. This is indicated by Hal Varian who has coined marketing as “the new finance”:

*“I think marketing is the new finance. In the 1960s and 1970s [we] got interesting data, and a lot of analytic fire power focused on that data; Bob Merton and Fischer Black, the whole team of people that developed modern finance. So we saw huge gains in understanding performance in the finance industry. I think marketing is in the same place: now we’re getting a lot of really good data, we have tools, we have methods, we have smart people working on it” - Hal Varian, 2007.*

The reason why online marketing can be seen as the new finance is due to following characteristics:

- Abundance of data
- Quantitative computer models of the data
- Real time data allows for continuous improvement

The argument is that data has enabled performance measurement and that one can more easily track, measure, assess and calculate the performance of the investments in online marketing. This accounting is one reason that search engine marketing is popular for businesses. The data has made it possible to precisely tell how much each click costs and what the performance of that click is. In offline marketing it is harder and more expensive to do the same accountancy, and it is therefore harder to know whether advertisers are getting ‘value for the money’. Businesses have always been investigating where their advertising money was spent most effectively, yet digitalization has drastically lowered the cost and made it easy to check which ads are performing well. Data mining, A/B tests, ROI, Conversion Tracking and Auctions are becoming common within online marketing. The table below summarizes the main differences between offline and online marketing.

<b>Offline Marketing</b>	→	<b>Online Marketing</b>
Intuition driven	→	Data driven
Estimation	→	Calculation
Budgets	→	Profit
Static	→	Dynamic
CPM	→	CPC/CPA
Target groups	→	Target individuals

TABLE 2: Differences between offline and online marketing



## 7. The Evolution of Search Engine Marketing

This section outlines the evolution of search engine marketing. We start out investigating the beginning of internet advertising and finish with search engine marketing as it is known today. This section is mainly inspired by Battelle (2005), Donaldson (2008) and Immortica (2007), who in-depth outlines the evolution of search engine marketing.

**The rise of portals:** In the beginning of the internet, the most common method to advertise was to pay a web portal to place a link to your website. At that point portals such as American Online (AOL) and Yahoo! could be characterized as the gatekeepers for businesses on the web. E-commerce companies paid large amounts to these portals to simply place a link to their businesses on the portal. There were no standard pricing arrangements and agreements were made through negotiations.



In **1994** the cost-per-mille (CPM) model was introduced on the internet. Advertisers at this point paid a flat fee to have their banner ad shown for a fixed number of times. CPM was pioneered on the internet by the magazine HotWired.com, which displayed banner ads for AT&T. When the user clicked on the banner, the user was directed to the website of AT&T. This event is known as a 'click through'. Around that same time, cookies were invented. Cookies are a consumer tracking technology that makes it possible to track visitors and measure how many times a webpage is viewed. Tracking cookies makes it possible to measure the number of impressions and click-throughs of an ad. Web sites also rely on cookies to customize what users see and to target ads precisely.

*"Cookies fundamentally altered the nature of surfing the Web from being a relatively anonymous activity, like wandering the streets of a large city, to the kind of environment where records of one's transactions, movements and even desires could be stored, sorted, mined and sold."* (Schwartz, 2001)

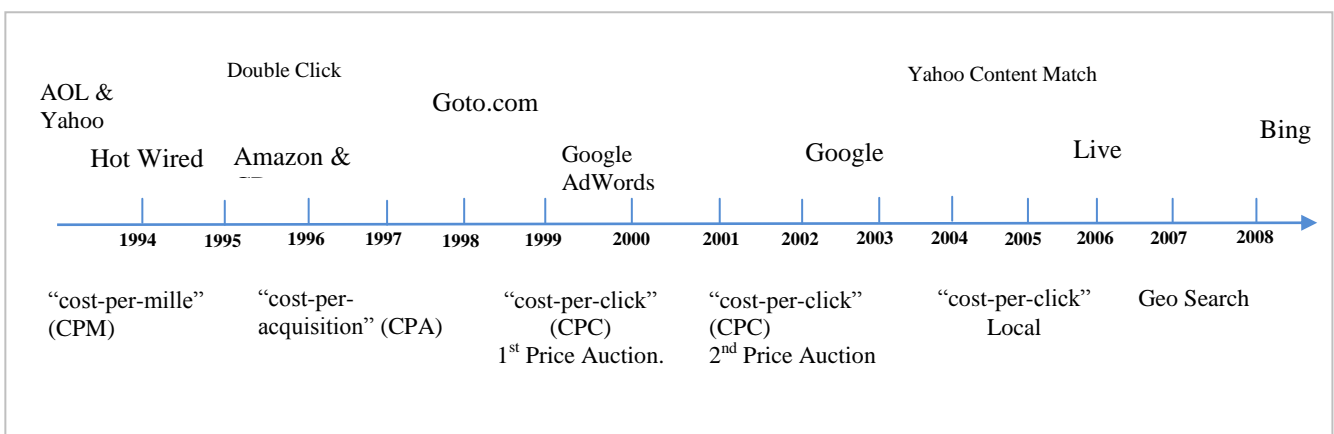
In **1996** cost-per-acquisition (CPA) was introduced by Amazon and the online music shop CDnow. It can be characterized as affiliate marketing where webpages have a link to a product at Amazon, and if someone buys the product, Amazon pays a percentage back to the original web page.

In **1998** the search engine Goto.com was introduced and offered advertisers an opportunity to bid in an auction on how much they would pay to appear as the top result on a search result page. Goto.com did not distinguish between organic search results and paid search results. Goto.com was therefore an advertising search engine where they auctioned off search results (and not ads which is the industry standard today). Furthermore Goto.com was the first to introduce cost-per-click (CPC) pricing. Advertisers could specify which keywords were relevant for their products, and how much they were willing to pay when a user clicked on their “paid-search-result”. The cost-per-click model at Goto.com was the first kind of advertisements on search engines where advertisers only had to pay when someone clicked on their ads. This differed from banner ads which were sold on a CPM basis. Therefore ads were no longer sold per thousands of impressions but instead one click at time.



One problem with the CPM model was that hosts of a website often did not care whether their users clicked on the ad, as long as they were paid by the number of impressions. Introducing CPC provided incentives for the host to improve the ad and therefore aligned the incentives of the host and advertiser. Cost-per-click is a performance-based model and can be seen as a middle ground between CPM and CPA. The first CPC ads at Goto.com were priced by a first-price auction.

In **2000**, inspired by Goto.com Google launched their advertising program named AdWords. Contrary to Goto.com, Google decided to separate the ads from the organic search results. Google AdWords was also auction-based and let advertisers deliver relevant ads targeted to search queries. In the beginning of the Google auction advertisers were bidding based on a CPM pricing model.



**FIGURE 9:** The Evolution of Search Engine Marketing

In **2001**, Goto.com, the frontrunner of search advertising, changed name to Overture. Overture means paid introductions and the company was acquired by Yahoo in 2003.

In **2002**, Google changed its pricing model from CPM to CPC and at the same time change the auction format from a first price auction to a second price auction. In section 8 we will investigate in more detail the specifics around the ad auction.

In **2003**, Google launched AdSense. Google AdSense is a program that enables web sites that are part of the Google Network to deliver ads that are relevant to content on web pages. This is known as contextual advertising. For example, if there is an article about computers in an online magazine, Google AdSense would display an ad for laptops. The idea is that users reading about computers might also be interested in buying a new computer. Contextual advertisements differ from ads on the search engine result page because it is not driven by intent-based queries of consumers, but rather by the content of a site, which determines which ads will be showed.

Later in **2003** after Google launched their AdSense program, Yahoo! followed with a contextual advertising program called Content Match. Both the Content Match and AdSense system have the similarity that they are working as an intermediary in a matching market where they match advertisers with content on web pages.

In **2004**, local cost-per-click was introduced. This is a service provided by search engines similar to what we as Yellow Pages. The rise of local search enabled that small businesses could easily reach a local customers.

In **2005**, Geo Search on Google Maps was launched which provided a geo search experience by combining maps, with yellow-pages listings. These also included ratings, reviews and other business information.

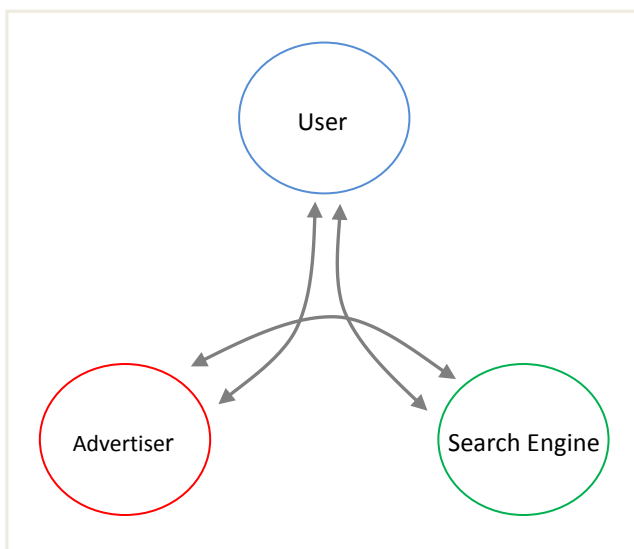
In **2006**, Microsoft search engine MSN Search, which was first launched in 1998 using results from Inktomi, was replaced by Live Search which was scaled up to compete with industry leaders Google and Yahoo.

In June **2009**, former Microsoft search engine (Live, MSN search) was renamed to Bing and several new features were added. This has been viewed as Microsoft's response to compete in the search business.

To sum up the chronology above, there are three major stages in the development of search engine marketing. First, ads were sold manually, slowly, and on a cost-per-impression basis. Second, Goto.com implemented keyword-targeted per-click sales and a first-price auction mechanism. Thirdly Google implemented selling ads by a second price auction, a procedure which was later adopted by Yahoo. It is interesting to note that all three pricing models, CPM, CPC and CPA are widely used on the internet. However the specific sector of search engines has converged to a cost-per-click pricing after several years of evolution.

## 8. Search Engine Marketing - The Fundamentals

This section examines the fundamentals of search engine marketing and is based on Goodman (2009). In search engine marketing three players interact: the user, the advertisers and the search engine. Each of the players has different goals. The primary goal for the user is that the search engine generates a good organic search result. The goal of the advertiser is to maximize the number of users that click on the ad and get directed to their web page. Lastly the search engine seeks to satisfy the users so they will return to the search engine, and also seeks to satisfy the advertisers so they will continue to advertise. We can investigate search engine marketing from the perspective of each of these players. It follows from the delimitation that we primarily focus on the relationship between the advertiser and the search engine. Therefore although the user is an important player, we will in the rest of the project focus on the advertiser and search engine relationship.



**FIGURE 10:** The Three players to satisfy in the Ad Auction. (AdWords Support, 2009)

### 8.1 Search Engine Marketing – from the viewpoint of advertisers

Search engine marketing is a method for advertisers to direct users to their websites. The mechanism of search engine marketing is that advertisers buy various keywords which are related to their product. Then when a user searches for the keyword that an advertiser has bought, the search engine will show an ad related to that keyword. Below we look at a practical example of search engine marketing.

Imagine that you sell laptops and you want to attract new customers. You would especially like to catch the attention of people searching for “laptops” in a search engine, as they could be potential customers. To achieve this, advertisers can buy the keyword “laptops” which implies that when a user searches for laptops it will trigger the ad. In the example in the figure on the next page you can see the Google search result page for a search on “laptops”. The organic search result is displayed to the left under the search query, and some relevant ads are displayed to the right. In this specific example there are three advertisers, which have bought the keyword “laptops”.

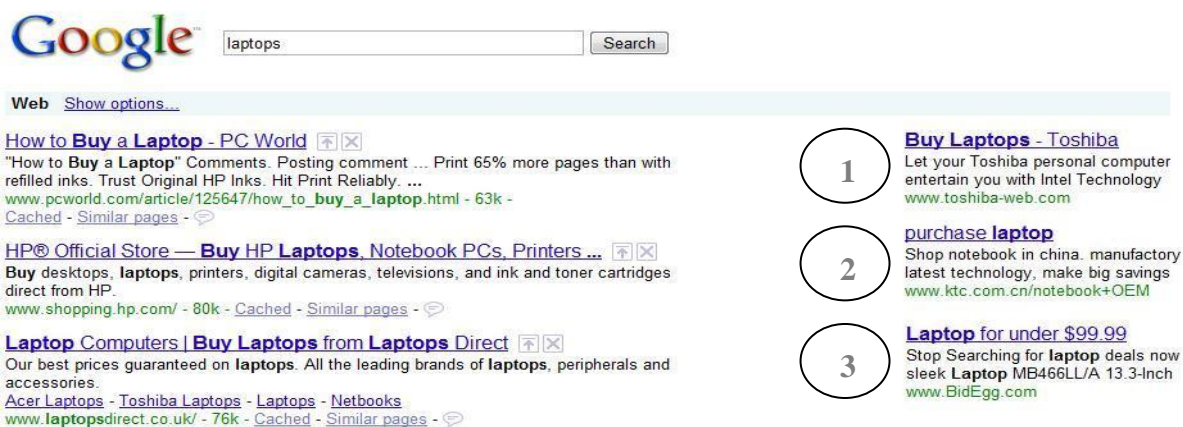


FIGURE 11: Google search engine result page - example.

## 8.2 Location, Location, Location

The placement of the advertisement on the search engine result page is a key factor determining the how much exposure and how many clicks an ad receives. The ad position which receives the most clicks is the first position in the top right corner. This is indicated by number 1 in the figure above. The next best ad position in terms of clicks is position number two just below number one, and so forth. There is a direct relationship between exposure and how many clicks an ad receives. However the ads on search engines are priced on a cost-per-click basis therefore we present the attractiveness of different positions in terms of clicks. The higher the ad is placed on the search engine result page the better is the position. Other things being equal, advertisers prefer higher positions. Due to this fact the first position is more valuable to advertisers than the second position. The search engine tracks how many clicks an ad receives within a given period. At the end of the period, the search engine bills the advertiser based on how many clicks the ad has received. This is the cost-per-click model mentioned earlier.

## 8.3 Keywords

One essential element of search engine marketing is the keywords and queries that users type in the search box. It is the keywords that advertisers purchase that trigger the ads. This means that if an advertiser only has bought the keyword “laptops”, his ad will not be shown when a user is searching for “computers”. If an advertiser wants his ad to appear both when a user searches for “computers” and “laptops” he must buy both keywords. Therefore the keywords are what matches advertisers with consumers.

Multiple advertisers may want to purchase and have their ad shown for the same keyword. In this case the search engines have to decide on how many ads to show and which ad should have the best position. This challenge is the essence of ad auctions which is addressed later.

## 8.4 Customization

Advertisers have many opportunities to customize their keyword campaigns e.g. by limiting their campaign to a chosen area or region. Assuming that one is only interested in potential customers located in Copenhagen one can target ads to the Copenhagen area. This means that ads will only be displayed if the users' IP address is from the Copenhagen area. This is an effective technique for advertisers to target a narrow segment of the market and limit the competition for certain keywords. Morkovich (2005) outlines endless opportunities in which one can customize search engine marketing campaigns. For example if one does not want to be associated with “expensive laptops” the queries containing the word “expensive” together with “laptops” can be sorted out. Also it might turn out that consumers mostly buy laptops during weekends. In this case advertisers can choose to only have their ads shown during weekends. This is beneficial if one has a limited budget and wants to have the most value for their money.

## 8.5 The Challenge

In essence search engines are match-makers. They match users that want to buy stuff with advertisers that want to sell stuff. We noted above that ads in position one receive more clicks than ads in position three. Therefore one question is, how does the search engine decide which advertisement gets allocated the number one spot? And what is the price of that spot? This problem is illustrated in the figure below in the case where there are three advertisers and three slots.

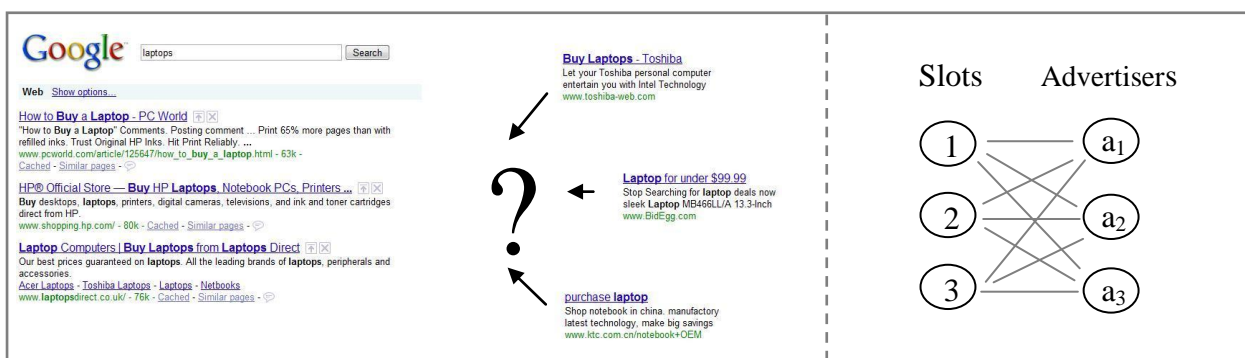


FIGURE 12: The problem of assigning slots to advertisers.

This allocation and pricing challenge can be addressed by the right market design. This is where ad auctions come into play. Ad auctions are the system that search engines use to price and allocate the different positions among advertisers. This is the focus of the next section.

## 9. Ad Auction Design

In this section we investigate how a search engine allocates the ad placements among advertisers and how they price different ad positions. Section 8 examined how advertisers can buy keywords and if a user searches for their keyword their ad will be shown. In fact, the way that advertisers buy keywords is by bidding in an auction. For each keyword, the advertiser submits a bid indicating what he is willing to pay-per-click, given that a user clicks on the advertisement. If the bid is high enough the ad will be shown. This is known as an ad auction. Below we consider the ad auction based on rules of the Google ad auction<sup>11</sup>. However the rules applied by Yahoo! and Microsoft do not differ substantially. The ad auction can be divided into two steps:

- Allocate the positions among advertisers
- Price the positions

The first step we denote the allocation scheme and the second step the pricing scheme.

### 9.1 Allocation Scheme

The allocation scheme determines the position of the advertisement on the search engine result page. For each advertisement, Google calculates an Ad Rank. The higher the rank the ad receives, the better position administered. From the advertisers' perspective, other things being equal, they prefer the highest possible rank to thereby maximize the number of clicks. The ad rank is a function of the advertiser's bid and the Quality Score. Quality Score is a measure of the relevance of the advertisement. The reason for a quality score is that ads from different advertisers have different quality, and thereby different probabilities of being clicked on by users, even if they are placed in the same position. To optimize the user experience Google has an interest in displaying the best quality ads to the user. For that reason Google favors ads of good quality and penalizes ads of bad quality. The advertiser with the highest ad rank will receive the highest slot in the search engine result page.

$$\text{Ad rank} = \text{bid} \times \text{Ad Quality} \tag{1}$$

The ad quality is computed by the search engine and is a product of several elements. Of these elements the predicted click-through rate is the most important. Predicted click-through-rate is the estimated probability by the search engine that the user will click on that advertisement. The bid

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<sup>11</sup> This section is constructed upon the rules in the Google AdWords auction. For complete details see: <https://adwords.google.com/support/aw/>



times expected click-through-rate can be interpreted as the ‘expected-bid-per-impression’. Expected bid per impression, it is how much it is worth for the advertiser to be shown on the search page. To optimize the user experience the search engine is interested in displaying the ads with the highest bid per impression on the highest rank. Also the relevance of the ad in relation to the keyword and the landing page has an impact on the ad quality.

## 9.2 Pricing Scheme

When Google has calculated the ad rank of the different advertisements and allocated the positions, they have to determine the price-per-click in each position. The advertisers’ price-per-click in the positions is not equal to the bids of the advertisers. Instead, in the Google ad auction the advertiser pays the minimum amount necessary for the advertiser to maintain their ad rank. This implies that for the advertiser in position one, we have to solve following equation to determine the price-per-click:

$$p_1 \times Ad\ Quality_1 = b_2 \times Ad\ Quality_2 \quad (2)$$

$p_1$  = price-per-click advertiser one

$b_2$  = bid-per-click advertiser two

When solving for  $p_1$  we find the price that advertiser 1 will have to pay to maintain his ad position.

$$p_1 = \frac{b_2 \times Ad\ Quality_2}{Ad\ Quality_1} \quad (3)$$

From here it follows that the price of the advertiser in position one becomes a function of the bid of advertiser in position two, and the ad quality of advertiser one and two. This means that ranking is not only a function of bids but is a multivariable of bids and ad quality. However if we assume that the ads have the same quality ( $ad\ quality_1 = ad\ quality_2$ ) we see that the price of the advertiser in position one is equal to the bid of the advertiser in position two.

$$p_1 = b_2 \quad (4)$$

Now we have established how ads are allocated to positions and how the price-per-click in the different positions is calculated. On the next page we illustrate this through an example.

### 9.3 Example

Assume that there are three advertisers and three positions on the search engine result page. Furthermore assume that the bids and ad quality is as illustrated in the table below:

Advertiser	Bid \$	Ad Quality	Ad Rank	Price-per-click \$
1	4	0.04	0.16 => 1	$(0.12/0.04) = 3$
2	6	0.02	0.12 => 2	$(0.05/0.02) = 2.5$
3	5	0.01	0.05 => 3	Min Price

TABLE 3: Example Google Ad Auction, (AdWords Support, 2009)

We see that although advertiser two has the highest bid he is not allocated to the first position. The reason is that advertiser one has a higher ad quality and his ad rank therefore becomes higher. We also see that the actual price-per-click is not equal to their bid, but is determined by the bid of the advertiser in the position below as well as the ad quality. In the example there are only three advertisers, and there is therefore no competition for position three. In this case advertiser number three only has to pay the minimum price determined by Google. Had there been four advertisers, the cost-per-click for advertiser number three would have been calculated in the same manner as advertiser number one and two. We also see that one way advertisers can improve the position of the ad is to increase their bid, which will improve the ad rank.

To conclude the allocation and pricing of ad positions can be summarized in the following steps:

- 1) Each advertiser submits a list of keywords, ads, and bids to Google.
- 2) When a user enters a query, Google compiles a list of ads whose keywords match that query.
- 3) The list of ads is then ordered based on the bids and the Ad Quality, which measure the relevance of the ad to the user.
- 4) The highest ranked ad is displayed in the top position, the second highest ranked ad gets the second best position, and so on.
- 5) If the user clicks on an ad, the advertiser is charged a price that depends on the bid and quality of the advertiser below him. The price charged is the minimum necessary to retain the advertiser's position in the list.

# Part II

The first part ended with an investigation of ad auctions which is the main focus of this project. Part II only deals with these ad auctions. In part II we present the theoretical foundation of auction and game theory and introduce and discuss a game-theoretic model of ad auctions. We end part II with a discussion of the challenges that search engines face when designing ad auctions.

The user is an important player in a larger perspective of search engine marketing. However, the investigation in part II is primarily concerned the relationship between the advertiser and the search engine.

Furthermore the ad quality is important when ranking ads, nevertheless, from now on, we shall treat the ad quality ‘as given’ – and only refer to ad quality in order to balance the conclusions.

## 10. Auction and Game Theory

In section 8 we outlined that one challenge search engines face is the allocation of ad positions on a search engine result page to advertisers. In section 9 we examined how auctions are used in practice to solve this task. This section establishes the key terminology of game theory and places ad auctions within the larger framework of auction theory. Auction theory allows us to examine in greater detail:

- A model of ad auctions
- Optimal and equilibrium bidding strategies in ad auctions
- The efficiency of the current ad auction design

Furthermore auction theory helps us explain why auctions are used as a pricing mechanism of ads, which we will return to in section 11. This section is also the foundation for a model of ad auctions, which we investigate in section 15. The overview of auction and game theory presented here is primarily inspired by Bierman and Fernandez (1998) and Krishna (2002).

### 10.1 Overview of Game Theory

From a search engine's perspective it is interesting to understand how advertisers bid in ad auctions. Similarly from the advertisers' perspective, an understanding of the optimal bid is important. In order to examine this interaction, auctions are typically studied through the application of game theory. Game theory is the science of strategy and attempts to logically determine the actions, that "players" should take to secure the best outcomes for themselves in a given "game". This can be compared to the study of a chess game where we have players, rules, tactics, winners, and so on.

Game theory within economics is concerned with how individuals' actions affect other individuals. It is possible to study auctions as a game, because we have clearly defined players, who are the advertisers, and concise rules for the game which are the auction rules. The reason we want to study auctions as a game is that bidding does not take place in a neutral environment. To illustrate this, think of the difference between the choices of a lumberjack and those of a general. When the lumberjack decides how to chop wood, he does not anticipate that the wood will fight back; his environment is neutral. But when the general is fighting an enemy's army, he must expect resistance to his plans. Like the general, a bidder in an auction must recognize his interaction with other intelligent bidders. Some of the useful terminology of game theory is presented below.

The individuals in a game are represented as **players**, and game theory is the study of how players take other players' actions into account, when making their decisions. In this process the players form **strategies** about how to play the game. A strategy is a complete plan for how to move at each step of the game. The different strategies players can choose are not all equally good. Some strategies are better than others, and we often distinguish between dominant and weakly dominant strategies. A strategy is **dominant** if it earns a higher payoff than any other strategy, regardless of what other players do. A strategy is only **weakly dominant** if it is always at least as good as any other strategy. In most games, there is often no dominant strategy, and in that case the players' best strategy depends on the strategy of other players. We say that players have a **best response** to the actions of other player.

We can study a specific game according to how the players' moves are conducted in time. **Simultaneous games** (or static games) are games where both players make their moves at the same time. The well known Rock-Paper-Scissors game is an example of a simultaneous game. **Sequential games** (or dynamic games) are games where players move in turn, and thereby have the possibility to observe the moves of other players before they act. Chess is an example of this type of games.

Another element when analyzing auctions as a game is how much information is available to the players. If a player has to take an action without knowing the previous action of another player, we are dealing with **imperfect information**. If the player knows all previous moves, we are in contrast dealing with **perfect information**. Perfect information differs from **complete information**, which requires that every player know the payoff and strategies of the other players.

Lastly when we investigate the outcome of the game, in formal terms we are studying different **solution concepts**. To understand this we look at the information the players have available, and what strategies they will follow given this information. The most common solution concepts are equilibrium concepts. There are numerous equilibrium definitions, but **Nash equilibrium** (NE) is important to mention here. To simplify, a Nash equilibrium is a stable state where no player in the game, can do any better, by adopting another strategy. Furthermore, each player expects that other players will adopt their Nash equilibrium strategies. In other words, each player chooses his best response, anticipating that other players will do the same.

## 10.2 Overview of Auctions

Auctions have been in use for many years. Roman soldiers used auctions to sell plundered loot when they returned from the battlefield. Today when most people think of auctions, they picture an auctioneer encouraging buyers to accept higher and higher bids, while selling objects like antique paintings. Recently web sites like EBay have made auctions more popular, by offering an easy system to auction off goods.

Formally, an auction is a system for allocating property based on price competition among buyers or sellers for the right to purchase (or sell) a good. McAfee and McMillan (1987) define an auction as “*a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants*”.

It is useful to quickly review the terminology of auctions. Auctions can be categorized as **open** or **closed**. In open auctions anyone can submit a bid, in contrast to closed auctions, where bidders must be invited or approved by the seller. The auction literature also distinguishes according to the amount of information that is revealed during an auction. In **sealed-bid-auctions**, bidders submit a single bid in secret to the auctioneer, and the bids and the winner are not made public until the end of the auction. This differs from **oral auctions** where bids are public, and everybody knows what others are bidding.

Additionally, bids can be **ascending**, **descending**, or **simultaneous**. If the bids are ascending they start out low, and steadily rise until only one bidder is left who wins the auction. On the other hand, if bids are descending, they start out high and descend until the first willing bidder is found, who wins the auction. Often there is a minimum bid requirements in auctions, which is referred to as the **reserve price**.

With respect to the value of the object on sale, we distinguish between private and common values. If the bidder knows his own value of the object at the time of bidding, we are dealing with **private values**. If we have private values it is assumed that bidders do not know the private values other bidders place upon the object. Often the value of the object on sale is unknown at the time bidding, but the value is nevertheless the common for all bidders, in this case we talk about **common values**. Common values are often the case when the object later can be resold at a market price.

The bidder who submits the highest bid wins the auction (unless it is an auction among sellers for the right to sell a good, then the winner is the lowest bidder). The price the winner pays differs according to the rules of the auction. In **first-price auctions** the winner pays a price equal to his own bid; this differs from **second-price auctions** where the winner pays a price equal to the second-highest bid submitted. One interesting feature of second-price auctions is that the bidder's payment is based only on the bids of others, and not on the winner's own bid. Auctions also vary according to the number of goods being sold. In a **single-unit auction** only one item is for sale and in **multi-unit auctions** several items are one sale. An example of single-unit auctions are art auctions where each painting being sold is unique. One example of multi-unit auction could be the sale of several identical bottles of wine.

### 10.3 Four Basic Auction Types

The auction literature generally centers around four auction formats: English, Dutch, Sealed-bid-first-price and Vickrey auctions.

**English auctions** are the most common type of auctions. An English auction is often used to sell art, antiques, wine and other goods. If there is a reserve price, bidding starts here; once one bidder has shown interest in the object on sale, the auctioneer solicits further bids, every time raising the price by a predetermined increment. This continues until there is only one bidder left, who wins the auction and pays the price of his own bid. It is therefore an open, ascending, first-price auction.

In **Dutch auctions**, the price starts out very high and the auctioneer then lowers the price until the first bidder is willing to "call" (bid). This bidder wins the auction and receives the item at the price where he made the "call". This auction type is common in places like the Dutch flower markets, hence the name "Dutch" auctions. Formally it is a descending bid, first-price, oral auction.

In English and Dutch auctions bidders receive information about other bids during the course of the auction. In **sealed-bid first-price auctions** this is not the case. In these auctions, each bidder submits a single bid in an envelope to the auctioneer. The bidder with the highest bid wins the auction and pays his own bid. Formally (as the name implies) it is a sealed-bid first-price auction.

In **Vickrey auctions** each bidder submits one bid in a sealed envelope, the highest bidder wins, and the price is equal to the second-highest bid submitted. Vickrey auctions are interesting due to their ‘truth revelation’ properties. They are truth revealing because bidders have an incentive to bid their true value for the object being auctioneered. This eliminates time-consuming strategic game play, and ensures that the item is sold to the bidder who values it the most. Formally a Vickrey auction is a sealed-bid second-price auction.

#### **10.4 Equivalent auctions**

Four different auction formats have been outlined above. From the perspective of the seller, it is interesting how these different formats perform. It has been shown in the literature that Dutch auctions and first-price-sealed-bid auctions yield the same outcome, because for every strategy in the first-price auction there is an equivalent strategy in Dutch auctions. There is also a performance relationship between English auctions and Vickrey auctions, although this relationship is somewhat weaker and depends on the specific environment.

#### **10.5 Multiunit auctions**

A **Vickrey-Clarke-Groves (VCG) auction** is a multi-item extension to the single-item second price auction. VCG auctions retain the incentive compatibility property of Vickrey auctions by charging each bidder the opportunity cost imposed upon other bidders. We will return to this auction format in more detail later.

#### **10.6 Efficient auctions**

An auction is said to be efficient if in equilibrium the object being auctioned is always won by the bidder who values it the most. Auctions often yield an efficient outcome and we prefer efficient auctions because they yield a useful allocation of the objects on sale.

An alternative to auctions might be a fixed priced. Fixed pricing are often inefficient because the price does not equalize demand and supply. This is for example the case for the pricing of concert tickets. Often the price of tickets is too low which result in long lines and black market sales. In this case the tickets are not sold to those who value the concert the most but instead to the fastest, most patient or the luckiest buyers. It also happens that the price of tickets is set too high. This is also inefficient because then we will have unoccupied concert seats even though there are buyers willing to pay for those seats.



## 11. Auctions - The Pricing Mechanism

In section 5 we discussed the special cost structure of search engines. It follows from this discussion that cost-based pricing is not a suitable pricing mechanism for keywords. On the other hand, auctions display some features which make them very useful for search engines to sell ads:

- Auctions are suitable when the seller is unsure how buyers value an object.
- Auctions are universal in the sense that they can be used to sell any object.
- Auctions are anonymous, which means that the identities of bidders play no role in determining who wins the object and who pays how much.
- Auctions secure an efficient allocation of the goods.
- Auctions can be automated.

These features are the reasons why auctions are employed on a large scale by search engines. There are literally billions of keywords, making it difficult for search engines to correctly price every single keyword. In pricing keywords, search engines have to take into account the dynamic nature of seasonality, trends and competition for all keywords. The search engine simply does not have this information. When search engines employ auctions to price keywords, they are using the market to overcome these challenges: the market gathers the information allowing them to pair advertisers with positions on the search engine result page. One necessary feature of auctions is that they can be automated. This is controversial in the media business, where many prices are negotiated. Negotiated prices do not scale for the Internet where pricing happens on a real-time basis. Automated auctions are an effective mechanism that pairs advertisers with the appropriate prices for their ads. Competition among advertisers ensures that the dynamic, dispersed information is constantly portrayed in the current price of a keyword.

By using an auction the search engines is basically saying: “We don’t know what the price is, but competition will set the right price”. The price of keywords differs substantially. Below you can see

an example of some of the most expensive queries. These prices were retrieved on 31 March 2009. Besides these expensive keywords, there are millions of keywords where there is no competition. These keywords can be bought at the reserve price set by the search engine.

Rank	Term	CPC
1	loan consolidation student loans	\$53.76
2	online life assurance quotes	\$53.30
3	accident no win no fee	\$53.27
4	get auto insurance online	\$53.02
5	cheap life assurance quote	\$52.58
6	tax attorneys los angeles	\$52.42
7	scottsdale dui lawyer	\$52.34
...	...	...
∞	∞	Minimum bid

**FIGURE 13:** Expensive keywords and queries. (Spyfu, 2009)

## **12. The Environment of Ad Auctions**

The market for internet advertising exhibits some unique features which we need to disentangle to understand ad auctions. This section investigates the special environment of online ad auctions. Bierman & Fernandez (1998) lay out that the auction environment consist of the population of potential bidders, the values these bidders place on the object being auctioned, their attitudes towards risk, and the information they possess about each other's valuation and risk attitudes. We will therefore examine online auctions according to these elements. This discussion is primarily inspired by Edelman et al. (2005). There are five primary characteristics which separate online ad auctions from other auction environments:

- Bidding takes place continuously
- Search engines are selling a floor of goods
- Advertisers' valuation of clicks stays roughly constant
- The unit of advertisement is hard to define
- Advertisers submit one bid for multiple objects

We will investigate each of these in turn.

### **12.1 Continuous Bidding**

The advertisers are bidding in continuous time and whoever is the highest bidder at a given time will have his ad shown in the first position. Correspondingly, the advertiser with the second-highest bid on a given keyword at some instant will be listed in the second position at that instant. At any time, the advertiser can manage and alter his bids according to what he observes in the market. Furthermore, other advertisers can revise their bids, and the order of advertisements and prices will change accordingly. Advertisers can employ automated robots in order to respond to bids as fast as possible. If we compare ad auctions to auctions on EBay, we see that buyers are also bidding in continuous time but the object at EBay is sold at a specific date. This is referred to as a "deadline" auction. One approach to alter the auction environment of ad auctions would be to set a random stopping time, so that the advertiser with the highest bid at that instant wins the auction. This is known as a 'candle' auction.

### **12.2 Floor of Goods**

Not only does bidding take place in continuous time but the object is also sold in continuous time. Search engines are basically selling a floor of goods – the product is always available and you can always buy it if your bid is high enough. This means that the product is never sold out. It is an

unusual feature but not a unique feature. The market for electricity, for example is also selling a floor of goods.

Another feature is that clicks/impressions cannot be stored and sold at a later time. If the search term has no bids, then no advertisements are assigned to the page, and the ad positions are wasted. This makes advertising placements a very perishable commodity, similar to products like airline seats or hotel rooms. This is one reason why the auction mechanism suits this environment: because it ensures that whoever is willing to pay the most at a given time will have the best placement.

### **12.3 Valuation of Ads**

Edelman et al. (2005) argues that profit-per-click stays roughly constant for advertisers, making each extra click exactly as valuable as the ones before it. From an advertiser's perspective, customer number 100 is just as valuable as customer number 99. Normally, the marginal utility of a good decreases as we consume more of that good. But advertisers are not buying resources like electricity or water, they are buying clicks, which they expect to translate into customers. A web store that receives 200 clicks earns twice as much as a web store that only receives 100 clicks. Clicks do therefore not have a marginal decreasing utility which means that the profit-per-click is constant. Moreover, the marginal cost of serving extra customers online approaches zero and the quantity increases. (Note that this can be questioned which we return to in section 16).

One reason search engines employ an auction, as mentioned above, is that the valuation of clicks is difficult. Clicks and impressions have a private value to advertisers. The assumption of private values is most plausible in the case where the value of the object is derived from the consumption of that bidder alone. This differs from common value where the object has a resale value. Implicit in the private value model is that no advertiser knows with certainty the values attached by other bidders, and knowledge of other bidders' values would not affect how much the object is worth to a particular advertiser. However, this is somewhat ambiguous, because in some cases clicks have a resale value. If you go to a search engine and type in "New York Hotels" the top results are remarketers. These are companies that are essentially arbitraging your desire to know more about New York hotels into possibly selling you a hotel room. Not any of these advertisers are selling hotel rooms; instead they aggregate demand and resell to a large network of businesses. You can basically say that they arbitrage the value of the click stream. The remarketers view the clicks as an investment which they can resell.

## **12.4 Unit of Advertisement**

How does one define a unit of advertisement? From the advertiser's perspective the relevant unit is how much revenue the ad generates. Pay-per-acquisition is one pricing model based on this approach as we outlined in section 7. From the search engine's perspective the relevant unit is how much it earns every time a user performs a search. In this case, exposure or impressions becomes the relevant unit of measurement. We delineated in section 9.2 that at some level the price is based on exposure/impressions because search engines are adjusting for expected click-through-rate when ranking bidders. This means that if your expected click-through-rate is twice as high as your competitors, you will only have to pay half the costs to maintain your position. One of the reasons we have seen a lot of innovation in payment methods is precisely because the unit of advertisement is hard to define.

## **12.5 One Bid – Multiple Products**

The fact that there are several positions on a search result page is an interesting feature of ad auctions. Each position leads to a different number of clicks, and therefore you could argue that each position is a different product. Taking this aside, the advertiser is still only submitting one bid, for all the different positions/products. This could be seen as an unusual bid requirement because different items are on sale, in the sense that position 1 is different from position 2. However the requirement makes sense in an environment where the advertisers value the clicks equally, no matter which position the clicks originate from. In that case the value of each position is proportional to the number of clicks in that position.

There is a technological limit of the number of ads which can be displayed on a search result page. For most search engines, the limit is eight to ten ads. This limit of available ad positions is referred to as the page inventory. You could compare the problem of selling the page inventory to the problem a shopping mall faces. A shopping mall also has to allocate different shops to different areas and price them accordingly.

## 13. Ad Auctions as a Game

In ad auctions there are clearly defined players and carefully followed procedures - this creates a foundation for studying ad auctions as a game. Since advertisers can change bids frequently, ad auctions can be thought of as a continuous game. When we study advertisers' bidding behavior in ad auctions, one important element to understand is whether truthful bidding is a dominant strategy. Our goal below is to investigate whether truthful bidding is a dominant strategy in ad auctions. We start out investigating this by examining the first-price ad auction which was the industry standard until 2002. Hereafter we explore the second-price ad auction which has been the industry standard since 2002.

### 13.1 First Price Ad Auction – before 2002

In section 7 we described how Goto.com (which later became Overture and was acquired by Yahoo in 2003) was the first search engine to sell ads through an auction mechanism. The auction was constructed as a first-price auction, where the advertiser who bid the highest price won the top position and paid his own bid. The second-highest bidder won the second-highest position and paid his own bid, and so forth. The ease of use, low entry costs, and transparency of the mechanism made this model popular. Though this model was conceptually straight forward, it also had some undesirable implications. To illustrate this, consider following example:

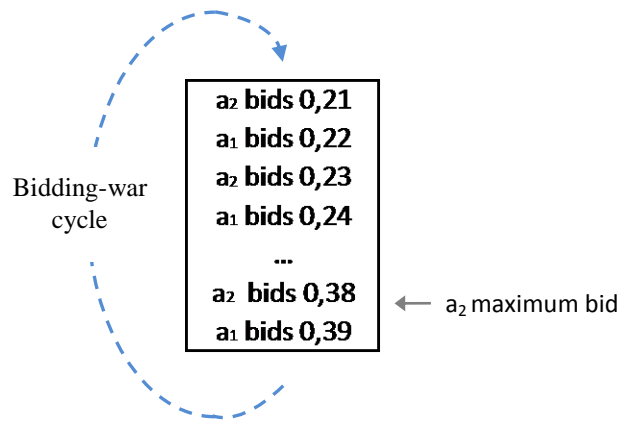
**Example 1:** Assume that three advertisers are competing for two positions. An ad in the first position receives 100 clicks per day and the second position 90 clicks per day. The three advertisers' value-per-click is respectively \$2, \$1,95 and \$0,20. This information is summarized in

the table to the right. It is easy to show that the two positions will be allocated to  $a_1$  and  $a_2$  because they are willing to pay the highest price and will outbid  $a_3$ . The price-per-click of the

Position	Advertiser	Cliks	Value
1	$a_1$	100	2,00
2	$a_2$	90	1,95
3	$a_3$		0,20

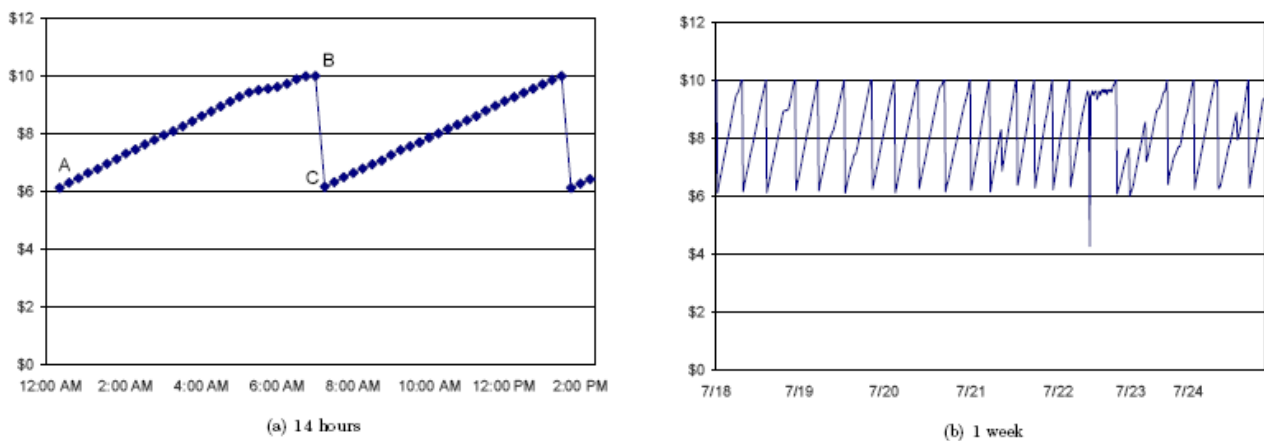
second position will be \$0.20 plus some small increment  $e$  - which is the minimum  $a_1$  and  $a_2$  will have to bid to outbid  $a_3$ . The price of position one is harder to determine. Imagine that  $a_2$  starts out bidding \$0,21 to occupy position two. Because position one receives more clicks than position two,  $a_1$  prefers position one but only wants to pay the minimum amount necessary to outbid  $a_2$ . The best move for  $a_1$  would therefore be to bid \$0,22 and occupy position one. Now  $a_2$  only has to bid \$0,23 to outbid  $a_1$  and occupy position one, and receive the 10 extra clicks. Clearly in this case  $a_2$  would

want to revise his bid to \$0,23 to get position one, because each click has a value of \$1,95 to  $a_2$ . This cycle would continue until the price-per-click of position one reaches \$0,39 at this point it no longer makes sense for  $a_2$  to bid higher because he can achieve a higher profit by lowering his bid to \$0,21 and receive fewer but cheaper clicks in position two (for complete calculations see appendix IV). At this point the bidding cycle starts over again,



because  $a_1$  is not interested to pay more than necessary to be in the first position and therefore lowers his bid to \$0,22. Clearly, there is no equilibrium in this game, and the best response for each bidder is to continuously revise their bids, when observing the bid of the other advertiser.

In practice this bidding game was often not conducted by humans but instead by programmed robots who were following prescribed bidding rules. Figure 14 below shows a real-life example of this bidding cycle which is represented by Edelman et al. (2005). It is based on data from Overture in 2002. The first figure presents the top bid for a specific keyword every 15 minutes during a two hour time period. There are two advertisers competing for the top spot. The advertisers start at point A below their maximum bids, from here they each in turn raises their bid by \$0,01 and outbid one another.



**Figure 14:** First-price ad auction bidding cycle, Data from Overture search engine 2002.

(Edelman et. al. 2005)

This continues until point B, where one of the advertisers has reached his maximum bid, at this point it is no longer profitable for him to raise his bid. Therefore he seeks to avoid overspending and lower his bid to 0,01 more than the third advertiser, this is shown in point C. The second advertiser observes this (or his autobidder does) and lowers his bid to \$0,01 more than the first advertiser – and the game starts over. The ‘sawtooth’ pattern is also represented in Figure B, which shows a continuation of this pattern for one week.

The example above shows that in the one-shot version of the game there is no dominant strategy equilibrium and it is quite unstable. The first price auction mechanism, in the environment of ad auctions was therefore non-optimal for three primary reasons:

- The best response for the bidders is to revise their bids as often as possible. This encourages inefficient investments in gaming the system.
- It produces volatile prices.
- The mechanism did not yield an efficient outcome. If the value of  $a_1$  is higher than  $a_2$ , the best solution for the search engine is that  $a_1$  is always allocated the number one spot. This was not the case in first price auctions.

### **13.2 Second Price Ad Auction – after 2002**

Google addressed these problems in 2002 when they introduced their second-price ad auction system. They realized that a bidder in position  $i$  would never want to pay more than a small increment,  $e$ , above the bid of the advertiser in position  $i+1$ . Google therefore adopted a second-price auction mechanism. It is interesting to note that this auction format was not invented by Google due to theoretic evaluations of different auction formats, but occurred by accident. As discussed above, the first-price auction was not attractive since bidders would want to reduce their bid to the lowest amount that would retain their position. The constant monitoring of the system put a significant load on Google servers, so they decided to automatically set the price to be equal to the second-highest bid - since this was what advertisers would want to bid anyway. The motivation for changing the format from a first-price auction to a second-price auction was therefore primarily due to technical and empirical observations. Fortunately, it turned out that the second price auction format was more stable than the first-price auction used before 2002.

As mentioned previously, second-price auctions have been of great interest to many game theorists due to their “truth revelation” properties. Furthermore, in the auction literature it has been widely demonstrated that bidding truthfully is a dominant strategy in Vickrey auctions, which are also a second-price auction. So one interesting question is whether the second-price ad auctions used by the search engines also have this tendency to reveal bidders’ true values. We can investigate this by an example.

**Example 2:** Imagine the following example with three advertisers competing for three positions:

Position	Clicks	Advertiser	Value	Bid	PPC	Profit
1	100	$a_1$	2,00	2,00	1,95	5
2	90	$a_2$	1,95	1,95	0,2	157,5
3	50	$a_3$	0,20	0,20	0,05	7,5

If  $a_1$  and  $a_2$  start out bidding their true values, the allocation of positions and pricing will be as shown in the table above. We see that  $a_1$  will be allotted the first position and  $a_2$  the second. Note, however, that the profit of  $a_2$  is higher than the profit of  $a_1$ . So in this case could  $a_1$  earn a higher profit by lowering his bid below  $a_2$ ? Suppose that  $a_1$  lowers his bid from his true value (\$2.00) to \$1,94. By doing this he will switch position with  $a_2$  and the new situation will be:

Position	Clicks	Advertiser	Value	Bid	PPC	Profit
1	100	$a_2$	1,95	1,95	1,94	1
2	90	$a_1$	2,00	1,94	0,20	162
3	50	$a_3$	0,20	0,20	0,05	7,5

In position two,  $a_1$  only receives 90 clicks, but the price per click is significantly lower and he therefore increases his total profit. In this case, tactical bidding is a better strategy than bidding the true value. Therefore, though we are dealing with a second-price auction, truthful bidding is not a dominant strategy.

The next question is of course what  $a_2$  will do when he observes the new situation. This question is answered in section 15 where I present a model of ad auctions.

Secondly it is important to note that the total revenue ( $PPC \times Clicks$ ) from the perspective of the search engine, decreases when  $a_1$  lowers his bid and exchange position with  $a_2$ . Other things being equal this behavior and strategy of advertisers is not optimal for search engines.



The current auction format for second-price ad auction seems non-optimal and unstable which fosters some interesting questions:

- Could search engines increase revenue by changing the rules of the auction?
- What is the best strategy for advertisers?

These questions will be answered in section 15 to 18.

## 14. The Rules of Ad Auctions

In section 9 we examined how ads are allocated to positions and how the price-per-click is determined. In section 10, we outlined a formal description of auctions. This section combines the two and lays out a formal description of the auction rules in keyword based advertising. The auction rules state who can bid, what bids are acceptable, how bids are submitted, what information is made public during the course of the auction, when the auction ends, how the winner is determined, and what price the winner pays for the item being auctioned off. In the context of online ad auctions, the buyers are the advertisers and the seller is the search engines. The product is ad placement and the corresponding impressions/clicks. A summary of the rules and characteristics of these ad auctions is outlined below.

**Acceptable Bids:** All bids are acceptable though there often is a *reserve price*.

**No. of bids:** All advertisers submit only one bid for each keyword. This is a *one bid requirement*.

**Units on sale:** Because there are several positions on sale, where each position is a different product, we are dealing with a *multi-unit auction*.

**Bidders:** Anyone can bid though the search engine though bidders have to go through some formal procedures to get approved. This means that the auction can be characterized as an *open* auction.

**Real time auction:** Bids are made *simultaneously* to the search engine (the auctioneer). The electronic system determines who the highest bidder at any given point is.

**Winner:** The winner of the auction is the bidder who submits the *highest bid*.

**Arrangement of ads:** Ads are arranged on the page according to the ranking of bids and higher placed ads receive more clicks.

**Pricing:** The ad auction is a *second-price auction* where the price paid equals the second-highest bid submitted. Moreover, we are dealing with “pay-per-click” pricing which means that the advertiser will only have to pay when someone clicks on his ad.

**Public information:** The advertisers can immediately see how their bid impacts the ad's position. Over time, the auction converges toward perfect information.

From the rules of the ad auction presented above we see that we are neither dealing with an English nor a Dutch auction. The reason is that ad auctions are second-price auctions. Furthermore ad auctions differ from Vickrey auctions because bidding takes place continuously and because it is a multi-unit auction.

From the advertiser's perspective, the rules of the ad auction are exogenously determined. The search engine, on the other hand, can change these rules to optimize the auction. We will return to this question in section 17, where we discuss some of the challenges that search engines are facing in relation to designing the rules of ad auctions.

## 15. Ad Auctions - A Model

In section 10, we outlined different common auction formats and in section 12, we concluded that the particular environment of ad auctions exhibit some unique features and is relatively new within the auction literature. Furthermore, we know from our investigations of ad auctions as a game in section 13 that truthful bidding is in some cases not the best strategy for advertisers. From this, it follows that although ad auctions are second-price auctions, the outcome differs from auctions of the Vickrey and VCG types.

Zhou and Lukose (2006) explain that one aspect that differentiates ad auctions from the most common types of auctions is the positional nature of ad auctions. Bidders are simply not bidding on one object, but on the position of their advertisement on the search engine results page. In a single-item second-price auction the highest bidder wins the auction but only pays the bid of the second-highest bidder. Position auctions in use by search engines extend the single-item second-price auction to a multi-item second-price auction. These issues raise some interesting questions:

- How does advertisers bidding affect each other?
- Can we find a strategy that advertisers should follow in order to maximize profit?
- How does bidding affect search engines' revenue?

To answer these questions, it is necessary to understand the behavior of bidders in ad auctions. To achieve this understanding, a formal model of the ad auction is presented. The model is based on our previous findings concerning the rules and environment for ad auctions. This model formalizes those rules.

The model outlined in this section is primarily inspired by Hal Varian's work on Position Auctions (2006a) as well as Edelman et al.'s (2005) work on the Generalized Second Price Auction (GSP).

In this section, we investigate a simple game-theoretic model of position auctions, and in section 17.1 we discuss position auctions in relation to VCG auctions.

### 15.1 Position Auctions – A Game Theoretic Model

Below a game-theoretic model of ad auctions is presented. A game-theoretic model is a mathematical game that represents a set of players and a set of actions available to each player. In this model the players are the advertisers and the actions are their bids. We can investigate players' expected profits, payoffs, from different strategies. The model represented details the problem of assigning advertisers to positions on a search result page, and how that assignment affects their payoff. First, I will present the nomenclature of the model.

#### The Model

Let  $S$  denote advertising slots on a search engine result page. We number them as:  $s=1, \dots, S$ . The total number of slots, the page inventory, is equal to  $S$ . If there are four positions then  $S=4$ .

Let  $x_s$  denotes the “click-through-rate” (CTR) for slot  $s$ . Because the CTR by definition is the percentage of clicks out of the total number of impressions, we can also think of  $x_s$  as the number of clicks in a given slot. We number the slots so that  $x_1 > x_2 > \dots > x_s$  which assumes that higher positions receive more clicks. Slot 1 therefore has a higher CTR than slot 2. This is consistent with our discussion in section 8 where we examined how higher-placed ads receive more clicks than lower-placed ads. We set  $x_s=0$  for all  $s>S$ . The logic here is that an ad will not be shown when  $s>S$ , and that the CTR therefore is zero for any ads not shown.

Let  $A$  denotes the total number of agents, who in this case are the advertisers. We number them as  $a_1, a_2, \dots, a_a$ . In our case the agents are the advertisers. We assume that the number of agents is greater than the number of slots,  $A>S$ . This assumption assures that there is competition for all slots on the search result page.

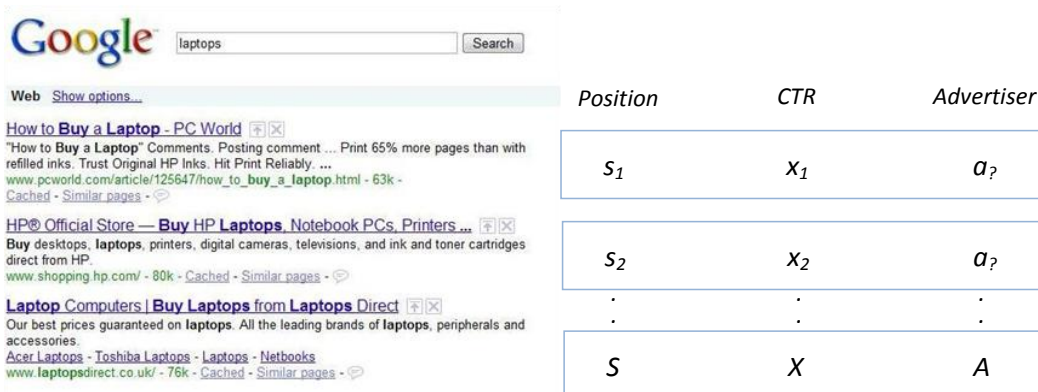


Figure 15: Formal representation of Positions, CTR and advertisers on a search engine result page.

Let  $v_a$  denotes the value-per-click for advertiser  $a$ . It can be understood as the expected profit per click, or the advertisers' maximum willingness to "pay-per-click". We assume that the value is positive,  $v_a > 0$ . This makes sense because if the value were negative then the advertiser simply would not advertise. We assume that the value does not change with positions. This means that an advertiser places the same value on a click no matter from which position the click originates.

Let  $u_{as}$  denotes agents  $a$ 's utility or valuation in a given slot  $s$ . The valuation is given by  $u_{as} = v_a x_s$ . This means that advertisers  $a$ 's valuation for a position  $s$  is equal to his value-per-click  $v_a$  times the number of clicks in that position,  $x_s$ .

Let  $b_a$  denotes the bid of agent  $a$ . Referring to the allocation scheme examined in section 9, the different slots  $s$  are sold via an auction where each advertiser bids an amount  $b_a$ . The slot with the highest click-through-rate,  $s_1$ , will be assigned to the advertiser with the highest bid, and  $s_2$  will be assigned to the advertiser with the second-highest bid.

Let  $p_s$  denotes the price of slot  $s$ . We are dealing with a second-price auction which means that the price for agent  $a$  in slot  $s$ , is equal to bid of the agent immediately below him. That is,  $p_s = b_{s+1}$ .

In the rules presented above, ads are only ranked according to bids. This is not precisely true. In section 9 we demonstrated that ads' ranking were according ad quality. Note that the rules presented in this model are therefore a simplification of the auction rules outlined in section 9. In the model we present here, we simply assume that all ads have the same quality. We can therefore ignore the quality effect, and rank ads only according to bids. If we drop this assumption it will get slightly more technical, the underlying findings remain unchanged.

Let  $\pi$  denote the profit which is given by:  $(v_a - p_s)x_s = (v_a - b_{s+1})x_s$ .

This formula describes that the profit of advertiser  $a$  in slot  $s$  is determined by his value-per-click  $v_a$ , minus the price-per-click in slot  $s$ ,  $p_s$ , times the number of clicks (CTR) in that slot. Since the price is determined by the bid of the advertiser in position  $s+1$  we see that the profit of advertiser  $a$  in position  $s$ , depends on the bid of the advertiser below him.

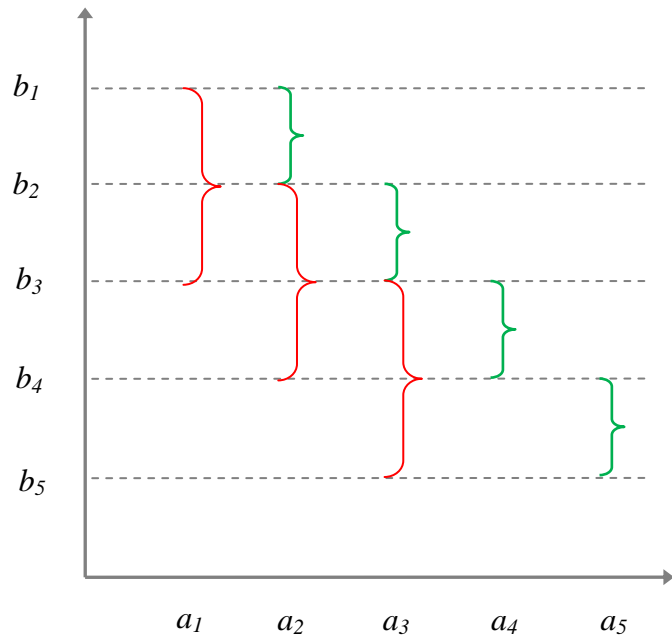
Table 4 below summarizes positions, values, bids and payments associated with an auction with  $S=4$  available slots.

Position	$CTR_{slot}$	Value	Bid	Price
$s_1$	$x_1$	$v_1$	$b_1$	$p_1 = b_2$
$s_2$	$x_2$	$v_2$	$b_2$	$p_2 = b_3$
$s_3$	$x_3$	$v_3$	$b_3$	$p_3 = b_4$
$s_4$	$x_4$	$v_4$	$b_4$	$p_4 = b_5$
$s_5$	$0$	$v_5$	$b_5$	$0$

**TABLE 4:** Bidding for ad positions (Varian, 2006a).

Position five is presented with a CTR of zero as it will not be shown because  $S=4$ , but it is the bid of the fifth advertiser that determines the price of position four. Remember that the CTR in the table is a representation of how higher placed ads receive more clicks, that is  $x_1 > x_2 > \dots > x_s$ .

Consider table 4 above. If we are analyzing the auction from a game-theoretic perspective, one question to ask is what an advertiser has to bid to move up or down one position. If  $a_3$ , who currently holds position  $s_3$ , wants to move up one position, he has to bid at least  $b_2$  plus some small increment  $\epsilon$ . This is represented by green braces in Figure 16. On the other hand if  $a_3$  wanted to move down one position he has to bid less than  $b_4$  but more than  $b_5$ . Since the price for  $a_4$  is  $p_4=b_5$ ,  $a_3$  only has to keep his bid higher than the price that  $a_4$  is paying to take over  $a_4$  position. This is represented by red braces in figure 16. From here follows that if an advertiser wants to move to a higher position, he has to bid higher than the *bid* of the advertiser currently in that position, but to move to a position below he has to bid higher than the *price* the advertiser in that position is paying.



**Figure 16:** Green braces = minimum bid to move up.

Red braces = max bid change to move down one position.

## 15.2 The Nash Equilibrium in Position Auctions

Given the rules and denotation presented above, we can investigate the outcome of the game. We model position auctions as a simultaneous-move game with complete information. The reason why we model positions auctions as a simultaneous (static) game is that although advertisers place their bids at different points in time, they have no information about the current bids of other advertisers; thus it is as if the decisions are made simultaneously. Each agent  $a$  therefore simultaneously chooses a bid  $b_a$ . The bids are then ordered and the price each advertiser faces is determined by the bid of the agent below him in the ranking. To model the game we assume that there is an equilibrium where prices stabilize. We place two restrictions on the equilibria of the game:

- No advertiser wants to exchange place with the advertiser below or above him.
- All advertisers play their static best responses

The above two restrictions satisfy the requirement for a Nash Equilibrium that no player can do any better by adopting another strategy; thus, in equilibrium, advertisers will prefer their current position to any other available position on the search engine result page. If an advertiser does not want to change his bid and shift position, we can say that the current bid is his best response. This assumption is reasonable because if an advertiser did not prefer his current position, he could simply lower or raise his bid and thereby move to another position. This equilibrium can be formulated as:

**Definition 1:** A Nash Equilibrium set of prices satisfies:

$$(v_s - p_s)x_s \geq (v_s - p_t)x_t \quad \text{for } t > s \quad (5)$$

*Profit current position*  $\geq$  *Profit in position below*

$$(v_s - p_s)x_s \geq (v_s - p_{t-1})x_t \quad \text{for } t < s \quad (6)$$

*Profit current position*  $\geq$  *Profit in position above*

Where  $p_t = b_{t+1}$

If we start out by examining the first inequality we see that the first term  $(v_s - p_s)x_s$  is simply the profit for the advertiser in his current position. The second term  $(v_s - p_t)x_t$  is the profit in the position below, because  $t > s$ . It is easy to see that if this inequality does not hold, we are not in equilibrium because the advertiser could benefit by lowering his bid and swapping positions with the advertiser below him. The same reasoning is true for the second inequality, in case the advertiser wanted to move up by one position. Therefore, these equilibrium rules ensure that an



advertiser is maximizing his profit, and thereby he does not have an incentive to change bid and position.

The rules for the equilibrium create a stable assignment. However, the equilibrium rules do not yield a unique outcome, but rather determines a range of bids which satisfy the inequalities. This is also reasonable because if an advertiser only changes his bid a small amount, compared to the different bids, it will not affect his position or payment. From here it follows that this stable assignment has some extreme points. It is interesting that, given  $v_s$  and  $x_s$ , we can solve the inequalities to find the extreme maximum and minimum points. It is the extreme points that determine the maximum and minimum equilibrium revenue attainable in position auctions. The minimum is preferred by advertisers and the maximum is preferred by search engines.

### 15.3 Symmetric Nash Equilibrium

The analysis of the equilibria is simplified if we only study a subset of the equilibria. Hal Varian (2006a) shows that in a Symmetric Nash equilibrium (SNE) of ad auctions following must hold:

$$v_{s-1} \geq v_s \tag{7}$$

$$(v_t - v_s)(x_t - x_s) \geq 0 \tag{8}$$

Inequality (7) shows that in equilibrium the value-per-click of an advertiser must be higher than value-per-click of the advertiser in the position below him. Inequality (8) shows that  $(v_t)$  and  $(x_t)$  must be ordered the same way as each other. This means that the positions with the highest  $x_t$  (CTR) will be assigned to the advertisers with the highest value-per-click. The outcome of the positions auction is efficient in the sense that the available ad positions are awarded to those who value them most highly. The outcome is also equitable in that the price an advertiser has to pay is determined by the other advertisers, – because it is a second-price auction.

### 15.4 Bidding

What happens to the advertiser in position  $s$  when the advertiser in position  $s+1$  raises his bid? We see that when an advertiser raises his bid, but does not change it enough to change position it will negatively impact the profit of the advertiser above. That is:

$$b_{s+1} \uparrow \Rightarrow \pi_s \downarrow \tag{9}$$

Naturally this alters the situation for the advertiser in position  $s$ , and if the advertiser in position  $s+1$  continues to raise his bid there will be a tipping point for  $a_s$  where the profit is higher in the position below. In that case he will lower his bid and the two advertisers will change position

By manipulation the inequalities of the symmetric Nash Equilibrium it can be shown that in one equilibrium the bid is given by.

$$b_s = b_{s+1}c_s + v_s(1 - a_s) \quad \text{letting } c_s = \frac{x_s}{x_{s+1}} \quad (10)$$

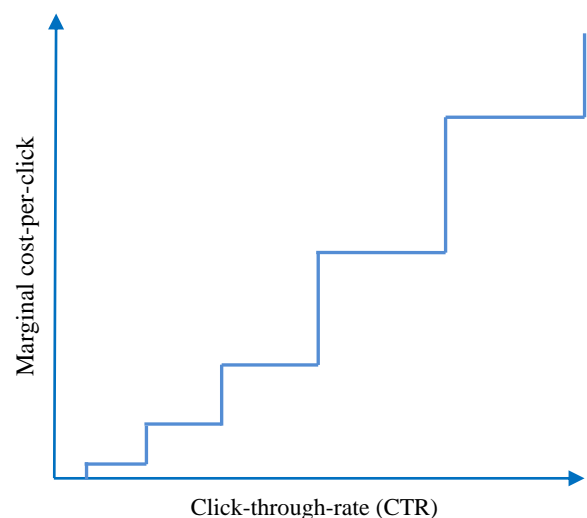
This makes sense because the inequality shows that an equilibrium bid is bounded by three elements:

- The bid of the advertiser below ( $b_{s+1}$ )
- The value of the advertiser ( $v_s$ )
- The difference in clicks between the two positions ( $c_s$ ).

Equality (10) can be interpreted as a bidding function. This also shows that position auctions are not dominant strategy solvable because the optimal bid depends on what other bidders are doing. In most games there is often no dominant strategy, and in that case the players' best strategy depends on the strategy of other players.

### 15.5 Supply curve of clicks

Our model shows that the symmetric Nash equilibrium creates an efficient outcome where advertisers who have the highest value-per-click are assigned to the highest positions. This indicates that in the equilibrium of position auctions, the marginal cost of clicks must increase as you move to higher positions. This has some implications for advertisers. Before an advertiser enters into an ad auction he knows that the number of clicks and the price of those clicks increase for higher-ranked positions. The higher his bid is, the more of the incumbent bidders will he displace. This means that the advertiser contemplating entering the auction is facing a 'supply curve of clicks'. In section 18, we discuss some practical bidding rules for advertisers as we return to the implications of this.



**Figure 17:** Supply curve of clicks. (Varian, 2006b)

## **15.6 Revealed Preferences**

Assuming that bidders are rational, in equilibrium, each bidder must prefer his current slot in relation to any other available slot. If this is not the case, the bidder can simply raise or lower his bid in order to receive another slot. This leads to a series of ‘*revealed preferences*’ relations from which we can uncover equilibrium bidding rules. For example, the advertiser in the first position must have a higher value-per-click than the advertiser in the second position, because otherwise the advertiser is not maximizing his profit. In this way, by observing bids on certain keywords, we can invert the bidding rules and find the values advertisers place on clicks.

## **15.7 Test of model against data**

The equilibrium outlined naturally has many limitations and rest on a couple of assumptions which I will discuss in section 16. Nevertheless, Hal Varian (2006a) presents some empirical results based on keyword bidding in Google ad auctions. In that case the model of position auctions describes bidding in real life ad auctions fairly well.

## 16. The Assumptions of Position Auctions

A model is a simplification of real life. The question is whether the assumptions made can be justified and how reasonable they are. In this section I focus on the implication of two assumptions.

These assumptions are:

- The model assumes full information
- The model assumes constant value of clicks

### 16.1 Complete Information

The Nash equilibria of position auctions assume full information. This implies that each bidder know the values of other bidders. This assumption differs from the majority of game-theoretic models of auctions, where one usually assumes that bidders' values are unknown but follow a probability distribution. Incorporating a probability distribution of bidder values into auction models makes it more complicated. Since one has to make additional assumptions regarding the distribution of values. In these models, bidder values typically falls into two categories:

- A private value model, where each bidder assumes that each of the competing bidders obtains a random private value from a probability distribution.
- A common value model, where each participant assumes that any other participant obtains a random signal from a probability distribution common to all bidders.

In private value models, one normally also assumes that values are independent across bidders. Often when modeling auctions one assumes symmetric bidders. This means the probability distribution from which bidders obtain their values are identical across bidders. In a private value model, which assumes independence, symmetry implies that the bidders' values are independently and identically distributed. In position auctions, we assume full information, including that the bidders' values are known. This simplifies the process because we ignore the-above complications with respect to distribution of values. Thereby the model of position auctions becomes less complicated.

However, one might question whether full information is a reasonable assumption. Clearly, when a bidder enters the auction he does not have full information about other bidders' values. Nevertheless, because the auction is continuous, as we examined in section 12, advertisers constantly observes the bids of other advertisers, and can infer their values. Therefore, although we model this as a static (simultaneous) game we know that in reality it is a dynamic game. This means that it is easy to experiment with bidding strategies in real-world ad auctions. Furthermore, several

third party software tools exist which are specialized to do this. Below is a snippet from the bid management company Iprospect's homepage:

*"...iSEBA is a true "agent" that tests each of the keywords on which you are bidding — every hour testing different positions within the search results, at different bid prices, at different times, and different days of the week — and then "learns" based on your results. iSEBA constantly uncovers the most productive keyword/position/price/time/day combinations on which to bid. From what it learns, iSEBA can be directed to maximize your campaign by your choice of factors, including: ROI, ROAS, conversions and traffic — for the best possible results"*

Also, Google reports click and impression data on an hour-by-hour basis and a few days of experimentation can yield good estimates of the number of clicks received for different bids. The easy experimentation with bids provides advertisers a good indicator of what other advertisers are bidding. This should indirectly inform bidders about the values of other advertisers. The availability of such tools, along with the ease of experimentation, suggests that the full information assumption is plausible. Combined with the fact that the assumption greatly simplifies the model, I find it a reasonable assumption.

### **Relaxing the assumption**

Although I find the full information a reasonable assumption one could relax it and model the game as a Bayesian game, where the characteristics of other players are incomplete. Hal Varian (2006) shows that you can model position auctions as a Bayes-Nash game, and that the equilibrium of position auctions is a generalization of the Bayes-Nash equilibrium of a simple auction.

Also, Edelman et al. (2005) demonstrate that position auctions can be modelled as a generalization of the standard English auction. In this case, bidder values are drawn from a continuous distribution function. They also show that there exists a Bayesian equilibrium in their generalization of the English auction.

## **16.2 Constant value of clicks**

The rules of the position auction require that for each keyword, bidders submit only a single bid, even though several different items are for sale: position 1 is somewhat different from position 5. The one-bid requirement makes sense in a setting where advertisers place the same value on a click no matter which position the click originates from. In this case, the value of being in a position is simply proportional to the number of clicks associated with that position. The model of position auctions therefore assumes that the value per click for advertisers is the same for all positions. This

value assumption is probably not sufficient to fully convey the reality: e.g., it does not allow for the possibility that users who click on position 4 are somehow different from those who click on position 2.

For many products, users research the market. In this case, they are likely to click on all of the top 3-4 ads, investigate the websites, compare prices etc. Some industry insiders argue that for consumer goods it is often better to be placed in position 3 and 4 compared to position 1 and 2, because users are more likely to turn into customers at this stage in the buying process. If this is the case, the value for each click is not the same across all positions, as the model assumes. Nonetheless, these limitations are probably not large enough to justify added complexity in the bidding language and the model.

Additionally, the model also assumes that the value-per-click stays constant when the quantity rises or falls. This means that the profit of an advertiser is proportional to the number of clicks. The model therefore assumes that the marginal value per click is constant. This assumption implies that more clicks are always better than fewer clicks. For some advertisers, this assumption is true. This is the case for many digital products like music or software. In these cases advertisers would be delighted to have twice as many clicks and sell twice as many products, because their value-per-click is roughly constant.

However, for many advertisers this assumption does not hold. The obvious examples are restaurants or concert providers. In these cases, advertisers clearly have short-term capacity constraints and the value-per-click changes as they approach their capacity. I do not believe above examples justify a more complicated model, but it is presented here to give an insight into the limitations of the model.

### **16.3 Other assumptions**

Naturally, the model makes many other assumptions, which also have been addressed in the literature. Below a quick review of some of them follows:

- **Advertisers are bidding on multiple keywords**

The model of position auctions reduces the complexity of the ad auction environment by focusing on one auction for one keyword. In reality, advertisers choose a single bid that will apply to many keywords. Hal Varian (2006) and Jansen & Mullen (2008) address these issues.

- **Advertisers have budget constraints.**

The model of position auctions does not take bidders' budget constraints into account. When advertisers have budget constraints they have to do a tradeoff between different keywords. Introducing budget constraints can help bring these kinds of tradeoffs into the model but it also changes the auction's properties. Abrams (2006) deals with these issues.

- **The quality of ads**

Different ads have different quality; some ads are more appealing to users than others. In section 9, we learned that advertisers were ranked according to bids and a measure of ad quality. The model of position auctions above does not take this into account. Abrams & Schwarts (2007) and Varian (2006) both introduce different models that address these complications.

- **Vindictive Bidding**

The equilibrium of position auctions assumes that an advertiser does not have an incentive to increase his bid if he cannot improve his profit. In keyword auctions, a keyword corresponds to a specific product or service where there are different providers who may be familiar with each other. In competitive markets, it can often be in a business's interest to try to squeeze other players out of the market, thus reducing market competition. This can be achieved by bidding higher to increase the cost of your competitors' ads and deplete their budgets. Zhou & Lukose, (2006) present empirical evidence which supports this type of bidding strategy, and present a model which incorporates this bidding behavior.

- **Brand Building**

Many campaigns are brand-building campaigns. If advertisers are not selling a product but instead building a brand, it is hard for advertisers to even know their own values when bidding.

## 17. Search Engine Challenges

Auction markets like electricity auctions or spectrum auctions have traditionally been designed by economists for specific settings. Ad auctions differ in this respect. The development of search engine marketing has shown that the rules for ad auctions have been an evolutionary process. In section 7, we outlined how online ads first were sold manually, slowly, and on a cost-per-impression basis. Then ads became keyword-targeted and were sold on a per-click basis. The pricing mechanism also developed from negotiated prices to automated first-price auctions - and today second price auctions are the industry standard. But what is the next step in this evolution and are the rules for current ad auctions optimal considering the environment?

Until now, we have taken the auction rules as given and simply investigated the outcome of ad auctions. The question could be reversed; given some advertisers and their values, is it possible to improve the rules of the auction? The search engine could change the rules regarding the reserve price, entry fee, invitation of bidders, closing rules, bidding increment or how bidders are allocated to positions. Some rules may be more efficient and profitable to the search engine than others. In other words, is it possible to design an auction format that would yield higher revenue to search engines? In this section I investigate some of the challenges that search engines face when deciding on the auction rules. I discuss two challenges in detail, and give a quick review of other interesting challenges at the end of this section. The two challenges of our focus are:

- Could search engines increase their revenue by changing auction format to VCG?
- How important is competition, the reserve price and page inventory for search engines' revenue?

### 17.1 Position Auctions & VCG

In section 10, we outlined that a Vickrey-Clarke-Groves (VCG) auction is a multi-item auction with truthful bidding as a dominant strategy. VCG looks similar to position auctions because both mechanisms set each bidder's payment according to the bids of others, and not based on the bidder's own bid. Secondly, both position auctions and VCG rank bidders according to their bids. For example, the highest bidder under both mechanisms is allocated to the best position. Nevertheless, VCG and position auctions differ when we take the payment rules into consideration. VCG is not a second price auction. Instead, pricing in VCG is equal to the opportunity cost each bidder imposes upon other bidders. The example on the next page illustrates this.



Position	Advertiser	Clicks	Bids	Position Auction			VCG		
				PPC	Payment SE	Profit Advertiser	PPC	Payment SE	Profit Advertiser
1	1	200	\$10	4	800	1200	3	600	1400
2	2	100	\$4	2	200	200	2	200	600
	3		\$2						

**TABLE 5:** Example of position auctions compared to VCG.

VCG charges a bidder the externality that he imposes on others, i.e. the decrease in the value of clicks received by other bidders because of his presence. In the example above advertiser 2 excludes advertiser 3 who bid \$2, who misses the opportunity of 100 clicks. Therefore the payment in position two is \$200 ( $2 \times 100$ ). Advertiser 1 excludes advertiser 2 (from position 1) who misses the opportunity of 100 extra clicks which each has a value of \$4. Advertiser 1 also exclude advertiser 3 from position two. In total his payment therefore is the cost he imposes on the two advertisers which is \$600 ( $100 \times 4 + 100 \times 2$ ).

From the example above, we see that the position auction yields higher revenue to the search engine than the VCG auction. On the other hand, the VCG auction has a stronger theoretical pedigree, including truth-telling as an equilibrium dominant strategy. Note that we examined in section 12 that truth telling is not a dominant strategy in position auctions. VCG therefore reduces the incentives for strategising and thus makes bidding more straightforward for the advertiser. The conclusion is that if the search engine were interested in changing the auction format to minimize strategic behaviour, VCG would be a good alternative, but the revenue in VCG might be lower compared to position auctions. In case a search engine would like to test the VCG format and study bidder behaviour, a practical implementation could simply be to let advertisers choose which auction format they prefer.

## 17.2 The Importance of Competition, Reserve Price and Ad Positions

The competition for a specific keyword determines whether all positions on the search engine page are sold. When all positions are sold, we say the page is oversold. Conversely when the page is undersold there is not competition for all positions. For example if there are only two advertisers competing for three positions the page is undersold. To investigate the importance of oversold and undersold pages, consider the two examples below. The examples are inspired by Varian (2006b).

### Undersold Auctions

If an auction is *undersold* the price of the last bidder is equal to the reserve price. To present this formally, we denote  $v$  as the value of each bidder and introduce  $r$  as the reserve price. We denote  $p_s$  to be the price in slot  $s$  and  $x_s$  as the number of clicks in slot  $s$ . Lastly  $x_m$  is the number of clicks in

the last slot. In equilibrium, each bidder has to be indifferent to the possibilities of either being in the last slot and paying the reserve price, or being in the slot above and paying  $p_s$ . If this is not the case, one of the bidders has an incentive to move to another position. To simplify the example we assume that each bidder has the same value  $v$ . We can then show the relationship formally:

$$\pi_1 = \pi_2$$

$$(v_1 - p_s)x_s = (v_2 - r) x_m$$

$$p_s x_s = v_1 x_s - v_2 x_m + r x_m$$

when  $v_1=v_2$

$$p_s x_s = v (x_s - x_m) + r x_m$$

We see that when the page is undersold the price of slot  $s$  is given by the expenditure for the last slot  $r x_m$  plus the incremental value of clicks in the position above  $v(x_s - x_m)$ . Note that if the page is undersold the reserve price is a key determinant for the price of all positions.

### Example 3 – Undersold ad auctions

Assume that there are only two slots and two competing bidders, and furthermore for a given time period  $r = 0,05$ ,  $v = 0,5$ ,  $x_s=1000$ ,  $x_m=900$ . We can then calculate  $p_s$ :

$$p_s 1000 = 0,5 (1000 - 900) + 0,05 \times 900$$

$$p_s = 0,095$$

Undersold auction			
Slot	clicks	PPC	Profit Search Engine
1	1000	0,095	95
2	900	0,05	45

In this *undersold* case the total profit from the perspective of the search engine will be 140.

### Oversold Auctions

If the page is *oversold*, the price of the last bidder is not determined by the reserve price but instead by the competing bidders – more precisely by the first excluded bidder with value  $v$ . The excluded bidder will drive the price of the last slot from the reservation price  $r$  to value  $v$ . Because we assume that all bidders have the same value it follows that each remaining bidder has to be indifferent to either being excluded or receiving the profit in the current slot – which is zero. From here follows:

$$(v - p_s)x_s = 0$$

$$p_s = v$$

#### Example 4 – Oversold Auctions

This means that with three competing bidders the price-per-click will be equal to the value  $v$  (0,5).

In this case, competition significantly increases the total revenue for the search engine. In the example, the revenue increases from 140 to 950 due to competition.

Oversold auction			
Slot	clicks	PPC	Profit Search Engine
1	1000	0,5	500
2	900	0,5	450

These two examples illustrate the importance of competition and that *oversold* pages are more profitable than *undersold* pages, not just because there are more bidders but because competition drives up revenue. What we also can infer from the above examples is that when there is no competition it is the reserve price that sets the level of prices for all positions. On the other hand, when there is competition it is the value of advertisers that determines the price of the different positions. This result is also intuitive, because as we outlined in section 10, an auction is a pricing mechanism based on competition among bidders. When there is not competition the outcome is not particularly good from the search engine's perspective.

#### Ad positions

One choice that search engines face is the number of ad positions on a search result page. There are a maximum number of positions which can be shown. This depends on the layout of the page and the size of the ads, etc., but the search engine could choose to limit the ad positions to only 3 or 4 ads per page. This would increase the competition because there are fewer positions. Our understanding of over and undersold pages can help us answer the question of how many ads to show on a page. More ad positions increase the number of ad clicks but also increases the risk of moving from oversold to undersold pages, which will reduce the price-per-click. Secondly, showing more ads might increase the risk of less relevant ads being shown which will diminish the user experience of clicking on the ads. Making the optimal choice depends on balancing these elements.

#### Reserve Price

The reserve price is also an important element in this context. In our example above, we saw how the reserve price determines the price if the page is undersold. In the case where there is only one bidder the auction mechanism does not work. The benefit of the reserve price in these cases is to increase search engines' revenue for ad positions with low competition. You can argue that the reserve price is a way to monetize the long tail of keywords.

On the other hand, the reserve price might also exclude some advertisers with a low value-per-click from bidding. A counterargument here is that if advertisers do not have a value per click above the reserve price (presently 0.01 cents at Google) they probably have a low-quality product which could degrade the user experience of searching.

### **17.3 Other challenges**

The above challenges are only some of the challenges that search engines face regarding the rules and design of auctions. Below is a short selection of other issues worth paying attention to.

- **Bidding language of the auction**

Currently search engines ask advertisers how much they are willing to pay-per-click. Other approaches could be to ask how many clicks they want. This would allow advertisers to reduce risk, as has been suggested by Immortica (2007) and Goodman (2009).

- **The product**

What is the right product to sell: clicks, impressions, acquisitions? Other formats of the product, for example video ads? This has been raised by Battelle (2005) and Immortica (2007)

- **Compensation of the user**

The user is the most important player in the market. We are in an economy of attention. Are search engines giving the users the right compensation (free services)? They could also give some kind of monetary compensation. This has been discussed by Immortica (2007).

- **How do ads influence each other?**

How do competitive ads influence each other or the user? This has been raised by Abrams (2007).

Though there are different possibilities for improving the auction rules, changing them might also result in a more complicated setting.

# Part III

The second part ended with a discussion of the challenges that search engines face in relation to ad auctions. The advertisers' perspective is presented in this Part III, which focus on one of the key challenges that advertisers face i.e. how much to bid for keywords? We are treating the rules of ad auctions as exogenously determined and present a practical bidding strategy to advertisers. This bidding strategy is consistent with our model presented in part II and is a recommendation to how rational advertisers should bid.

To finish, this third part ends with a summary of the conclusions and an outlook into the future of search engines and ad auctions.

## 18. Practical Bidding for Advertisers

All advertisers entering into ad auctions face the question of how much to bid for keywords? We know that the bid determines the ad position, and that the number of clicks differs with positions. This means that advertisers face questions like: If I increase my bid, how many clicks can I expect to receive? How much would those clicks cost? Should I bid higher? Today many advertisers are bidding based on goals such as “We want to be in the first position!” or “Our budget is \$1000 what is the maximum number of clicks we can get?”. These approaches to bidding do not yield an optimal outcome. Furthermore in relation to our model of position auctions these bidding goals violate the assumptions of rationality on part of the bidders. In this section I present a framework that helps advertisers optimize their bidding. I call this strategy ‘marginal bidding’.

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### 18.1 Marginal Bidding

We know from our model of position auctions that advertisers face a ‘supply curve of clicks’. A higher bid results in more clicks but also a higher price-per-click. This means that the supply curve is upward sloping. Moreover, advertisers know that the elasticity of supply differs from keyword to keyword depending on competition. In order to bid optimally these elements have to be taken into account. Demand and supply determines the market price but often we do not know how these curves look. Recently, online marketing has been fuelled with data which has made it easier to estimate the supply. In August 2009 Google released a tool named “bid simulator”. The bid simulator estimates the number of clicks different bids would receive. The estimation is based on historical data. Obviously, the bid simulator cannot

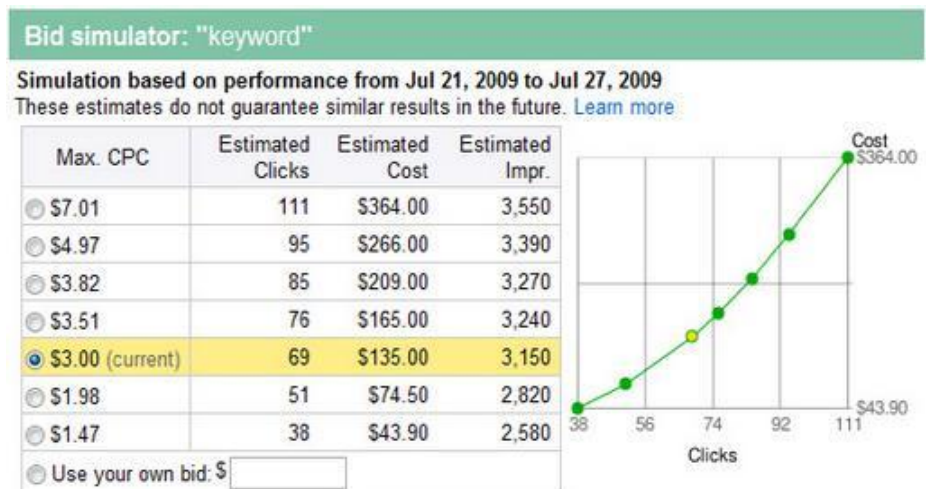


FIGURE 18 – Bid Simulator, cost curve of clicks. (Schwartz B. , 2009)

predict the future, but it gives an understanding of the tradeoffs advertisers are facing. In economics, we know this as the marginal trade off, which is consistent with the model of position auctions presented earlier. If we assume that an advertiser is actually facing the supply curve represented by the bid simulator in figure 18, what should he bid?

First of all, the goal of a rational advertiser is to maximize profit. In order to maximize profit we need to do marginal calculations. The idea is that when an advertiser moves from one position to the next, it not only increases the price-per-click for the extra clicks he receives, but increases the price-per-click for *all of his clicks*. We therefore need to take marginal-cost-per-click (MC) into account. Below is marginal-cost-per-click calculated based on data from the bid simulator.

Position	Bid	Clicks	Cost	CPC	Click value	MC	Revenue	Profit
1	7,01	111	364	3,3	5	6,1	555	191
2	4,97	95	266	2,8	5	5,7	475	209
3	3,82	85	209	2,5	5	4,9	425	216
4	3,51	76	165	2,2	5	4,3	380	215
5	3,00	69	135	2,0	5	3,4	345	210
6	1,98	51	74,5	1,5	5	2,4	255	181
7	1,47	38	43,9	1,2	5	1,2	190	146

FIGURE 19 – Profit calculations

We see that the cost-per-click in position 5 is equal to \$2. Note that cost-per-click is an **average measure**. Let's assume that an advertiser is contemplating moving to position 4 which would result in 7 extra clicks (76-69). How much would those extra clicks cost? Marginal cost-per-click is the increase in total cost divided by the increase in clicks. Total cost rises by \$30 (165-135). This

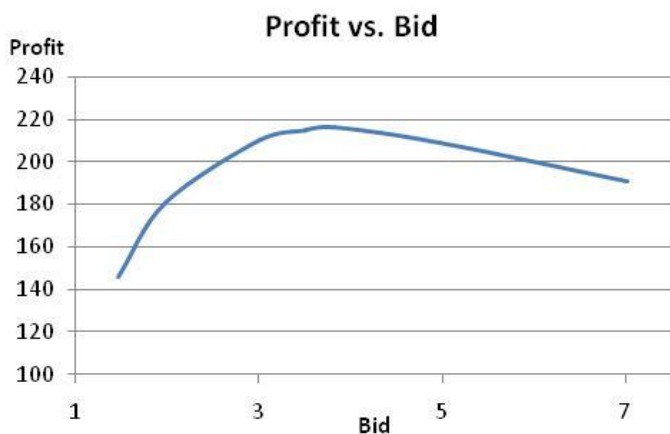


FIGURE 20 – Bid in relation to profit

means that you are paying \$4,3 per click (\$30 / 7 clicks). This is the **marginal-cost-per-click**, which gives a much clearer representation of the cost of moving up one position compared to cost-per-click which is the average measure. By using marginal-cost-per-click calculations we can find the optimal bid.

If we assume that each click has a value

of \$5 to our advertiser, we see that profit is maximized in position 3 where  $MC = MR$ . The estimated corresponding bid in position 3 is \$3,82. In the figure above we see how profit is related to different bids. Of course this is the bid of another advertiser currently in that position, and he will move down one position if our bid is \$3,82.

The above calculations are not difficult to carry out and when the data is available they could be set up automatically. An advertiser might be tempted to think that because his value-per-click is \$5 and the cost-per-click in the first position is only \$3,3 the first position will be the most valuable. However, as the example illustrates when you take marginal costs into account and focus on maximizing profit it is clearly not optimal to be in the first position. Therefore when bidding average cost (CPC) is an illusionary measure.

What the example illustrates is that explicit goals like “We want to be in the first position!” are inconsistent with maximizing profit, because such goals do not take marginal cost-per-click into account. Moreover, traditional marketing is used to “budget thinking” but bidding on an ad position according to a fixed budget is also inconsistent with profit maximization in these auctions. Instead, it is a better strategy for advertisers to determine how they value each click, examine which position is optimal given this, and then set a budget in order to achieve this.

We learned from our discussion of oversold and undersold pages that when there is less competition the price is also significantly smaller. This means that a good bidding strategy would be marginal bidding on less-competitive keywords.

## **18.2 Competitive Bidding**

We have discussed several times that bidding in ad auctions does not take place in a neutral environment. When advertisers change bids they must expect competitors to take actions in response to this. In this setting, there are naturally several competitive bidding strategies. One of them is called “gap jamming”, where advertisers raise their bids to a point just below their competitors’ so that the competitors will pay the maximum amount and more quickly deplete their budget. These tactics are complicated and often not credible. The advertiser conducting this strategy might also risk credibility and image if his strategy is disclosed. In general, I will not recommend these strategies.



## **19. Search Engine Marketing in Practice**

Though search engine marketing is becoming increasingly important it is still a relatively new field within marketing. In this section I will briefly present some of the practical challenges of search engine marketing. This is primarily based on my own professional experience. Lastly, I end this section by giving my recommendations to advertisers managing search engine marketing campaigns.

### **19.1 Explaining Search Engine Marketing**

Search engine marketing has to be seen in the broader landscape of TV, print and other online marketing activities. Most advertising campaigns are managed by third-party companies such as media agencies which play an important role in relation to search engine marketing. But they often face a challenge of explaining what search engine marketing is. This is primarily due to two reasons:

- Search engine marketing is dynamic
- Search engine marketing is technical

Advertisers are used to more traditional media like television and print. In these media, ads are sold on a fixed price on a cost-per-impression basis. This is not the case for search engine marketing. There is no fixed price and ads are sold on a cost-per-click basis, in an environment that changes in real time. You have to bid on thousands of keywords and constantly track their performance.

Secondly, online marketing has become technical. Understanding HTML, tracking scripts, and URL's are essential in order to maximize the performance of your online campaigns. Moreover, search engine marketing exists in close partnership with a new technical discipline called Search Engine Optimization (SEO). Search Engine Optimization is concerned with improving the organic search results for businesses. The dynamic and technical aspects have made it complicated to pitch and explain search engine marketing.

Sometimes, therefore simply explaining what search engine marketing is, can be hard. And clearly one of the disadvantages of the ad auction mechanism is that the pricing mechanism is hard to communicate to potential customers. These challenges have to be overcome before engaging in advanced bidder strategies.

## **19.2 Practical Recommendations to Advertisers**

Based on our previous findings, how should advertisers maneuver in the field of search engine marketing? Below I present four rules of thumb which should help advertisers govern their search engine marketing activities.

### **1) Organize the surroundings**

Sometimes the quality of a website is so poor that it is destroying the image of a business. At the same time, that company may be spending large sums of money on search engine marketing to direct customers to that website. This is non-optimal and therefore the first rule is, that advertisers should “organize the surroundings” before engaging in search engine marketing.

### **2) Let data drive decisions**

One of the advantages of online marketing is that data gathering is easy. Furthermore, you can change bids, campaigns, and keywords in real time. You can leverage this to make better decisions. If you are unsure how competitive a keyword is or how your ad will perform, you can easily do split tests and let the data solve the problem. Data-driven decisions are often better than intuition.

### **3) Be cautious with budgets**

Spending on search engine marketing is nowadays controlled by budgets. This fosters box thinking and does not acknowledge the dynamics of the environment. Advertising on search engines is an investment where advertisers should expect a future payoff. Therefore marketing spending should be governed by profitability analyses and not budgets.

### **4) When bidding do marginal calculations**

When bidding in ad auctions, advertisers should be aware of the marginal cost of clicks. Higher positions do not only yield more clicks but also more expensive clicks. The explicit goal should be to maximize profit and not maximize clicks. This can be achieved by applying a marginal bidding strategy as presented in this work.

## **20. Conclusion**

The following is summary of the conclusions throughout the project. They are presented in the same order as they were developed in the project.

### **Search engine marketing**

In order to understand search engines we begun by investigating the business model. We found search engines are intermediaries between users, advertisers and content providers. They mediate information and services for consumers, and derive value from those services using the traditional revenue streams of the media business – advertisements and subscriptions. We concluded that for Google, which is the largest player in the industry, 98% of the revenue is generated from advertisements. Finally, it was discussed how digitalization and the internet have facilitated easy data gathering regarding the performance of advertisements.

Considering that the main revenue source is advertisements we focused on how this ad revenue is generated. We established that search engines display ads next to the search results. The advertiser pays for these ads on a cost-per-click basis. This is known as search engine marketing. We studied how the placement of the ads is a key factor in determining how many clicks an ad receives. The position that receives the most clicks is the first position followed by the second and third. Due to this, higher positions are more valuable to advertisers. The scarcity of positions and how to price the positions can be addressed by the right market design. This is where ad auctions come into play.

### **Ad auctions**

We examined how search engines employ an auction system to price and allocate ad positions among advertisers. The basic design of ad auctions is that advertisers choose a set of keywords related to their product. Each advertiser states a bid for each keyword. The ads are then ranked by bids and ad quality. The ad with the highest ad rank will be allocated to the top position which receives the most clicks. The advertiser's cost-per-click depends on the bid of the advertiser below in the ranking.

In part two we started out by investigating auctions in a larger framework. We concluded that an auction is a reasonable pricing mechanism due to the dynamic environment of search engine marketing. We outlined how auctions mainly come in two categories: a first-price auction where the

winner is the highest bidder and the price is his own bid, and a second-price auction where the winner is also the highest bidder but the price is the second-highest bid. The current format of ad auctions has been an evolutionary process. First, ads were sold in a first-price auction and later the format was changed to a second-price auction. We demonstrated that the first-price auction was not attractive to the search engine, since bidders wanted to reduce their bid to the lowest amount that would retain their position.

### **Position auctions**

Next we discussed the environment of ad auctions. We presented how bidding is conducted continuously and advertisers' value per click is can be considered constant. We investigated how the current format of ad auctions can be modeled as position auctions. The position auction is a second-price auction specific to the environment of search engines. We presented that in position auctions bidders are not bidding on one object, but rather on the position of their ad on the search engine result page. In order to examine which challenges arise in relation to ad auctions, we presented a game-theoretic model of position auctions. We placed two restrictions in order to construct equilibrium of position auctions where prices stabilize. These two restrictions are:

- No advertiser wants to exchange place with the advertiser below or above him.
- All advertisers play their static best responses

We concluded that these equilibrium rules create a stable assignment. However the equilibrium rules do not yield a unique outcome, but rather it determines a range of bids which satisfy the equilibrium rules. We discussed how position auctions do not have equilibrium in dominant strategies furthermore truth-telling is not equilibrium of position auctions. Nevertheless, the model showed that the outcome of the ad auction is *efficient* in the sense that the available ad positions are awarded to those advertisers who value them the most. The outcome is also *equitable* in the sense that the price an advertiser has to pay is determined by other advertisers. From the model of position auction we also found that advertisers are facing a 'supply curve of clicks'. This means that the cost-per-click for ads increases as the ad position improves.

In order to get the full picture of position auctions we hereafter discussed the underlying assumptions. We primarily addressed the assumptions of complete information and the constant value of clicks. Though these assumptions are violating some of the properties of real life auctions we deemed both of them to be reasonable. This is due to the environment of position auctions and

the fact that it would complicate the model if these elements were incorporated and taken into account.

Based on our conclusions on position auctions we investigated whether search engines could increase revenue by changing auction format to a Vickrey-Clarke-Grove format. We showed in a simple example that position auctions yield higher revenue to the search engine compared to the VCG auctions. However the VCG auction has a stronger theoretical pedigree, including truth-telling as an equilibrium dominant strategy. We also examined the importance of competition and that *oversold* pages are more profitable than *undersold* pages. This is due to the drivers of competition. Without competition the price level of ad positions are determined by the reserve price.

### **Ad auctions in practice**

In the third part we took a practical approach to ad auctions based on our previous findings. We argued that search engine marketing is hard to explain to potential customers due to its technical and dynamic nature - and that the pricing mechanism is rather complicated. We kept in mind that advertisers face a ‘supply curve of clicks’ and discussed how explicit advertiser goals such as “We want to be in the first position” are inconsistent in a setting where you seek to maximize profit. Moreover “budget thinking” fosters an unhealthy environment where focus is on budgets and not on profits. In order to better navigate this area we presented four practical recommendations:

- 1) Organize the surrounding
- 2) Let data drive decisions
- 3) Be cautious with budgets
- 4) When bidding, do marginal calculations

The overall challenge for advertisers when engaging in search engine marketing is how much to bid for keywords in the ad auction. To address this challenge we ended this project by presenting a practical bidding strategy to advertisers. The strategy is titled marginal bidding and is the final recommendation to advertisers.

## 21. Outlook

What is the future of search engines and ad auctions? Search engines are a new business segment and are as such still underdeveloped. Search engines are still in the “text age” and have not yet figured out how to search music, games, videos and other content. The economic importances of search engines will increase as the technology improve. With this in mind I believe the importance of search engines will continue to increase during the next decade. However, this will not continue in perpetuity. Search engines are tied to the technology and will change alongside of it. Search engines therefore face a limited time-span and will be replaced as technology change and new business models emerge. They will one day suffer just as newspapers and TV networks suffer today.

With respect to the auction mechanism, ads in traditional media such as radio, newspapers and TV are today still sold manually in negotiated contracts. These media will sooner or later become fully digitalized and merge with the internet. When this happens will we still manually negotiate prices? Most likely we will not. When a business gets digitalized, fragmented and connected to the internet the pricing mechanisms will change in accordance with the environment. Auctions may be that new pricing mechanism. I predict three tendencies within this area:

- Continuation of digitalization and fragmentation of media.
- A rise in computer mediated transactions
- Automation of pricing and allocation mechanisms

One example of this happening is YouTube where the content is digitalized and fragmented. Recently YouTube launched the possibility of video advertisements. How could one manually price billions of video ads on YouTube? This is complicated and therefore these ads are also sold through an automated second-price-auction. In addition, social networks are also turning to the media model and automated auctions to sell ads. This following job listing from Facebook makes the point:

*“Facebook is seeking an advertising auction expert(...) The position will be responsible for shaping our rapidly growing online advertising market and adapting existing auction mechanisms to Facebook's unique environment”*

**Facebook 2009**

In contrast to search engines auctions are universal and are not tied to an environment. The use of auctions may increase in the future because they can be applied in any business setting looking for an efficient pricing and allocating mechanism.

## 22. Appendices

### 22.1 Appendix I: Google Financial Overview

	Year Ended December 31,				
	2004	2005	2006	2007	2008
	(in thousands, except per share amounts)				
<b>Consolidated Statements of Income Data:</b>					
Revenues .....	\$3,189,223	\$6,138,560	\$10,604,917	\$16,593,986	\$21,795,550
Costs and expenses:					
Cost of revenues .....	1,468,967	2,577,088	4,225,027	6,649,085	8,621,506
Research and development .....	395,164	599,510	1,228,589	2,119,985	2,793,192
Sales and marketing .....	295,749	468,152	849,518	1,461,266	1,946,244
General and administrative .....	188,151	386,532	751,787	1,279,250	1,802,639
Contribution to Google Foundation .....	—	90,000	—	—	—
Non-recurring portion of settlement of disputes with Yahoo .....	201,000	—	—	—	—
Total costs and expenses .....	2,549,031	4,121,282	7,054,921	11,509,586	15,163,581
Income from operations .....	640,192	2,017,278	3,549,996	5,084,400	6,631,969
Impairment of equity investments .....	—	—	—	—	(1,094,757)
Interest income and other, net .....	10,042	124,399	461,044	589,580	316,384
Income before income taxes .....	650,234	2,141,677	4,011,040	5,673,980	5,853,596
Provision for income taxes .....	251,115	676,280	933,594	1,470,260	1,626,738
Net income .....	\$ 399,119	\$ 1,465,397	\$ 3,077,446	\$ 4,203,720	\$ 4,226,858

#### Google Inc. (NasdaqGS: GOOG)

Last Trade:	<b>438.17</b>	Day's Range:	<b>N/A - N/A</b>
Trade Time:	<b>Jul 15</b>	52wk Range:	<b>247.30 - 537.05</b>
Change:	<b>0.00 (0.00%)</b>	Volume:	<b>2,948</b>
Prev Close:	<b>438.17</b>	Avg Vol (3m):	<b>3,357,430</b>
Open:	<b>N/A</b>	Market Cap:	<b>138.43B</b>
Bid:	<b>437.61 x 200</b>	P/E (ttm):	<b>32.03</b>
Ask:	<b>439.00 x 100</b>	EPS (ttm):	<b>13.68</b>
1y Target Est:	<b>464.82</b>	Div & Yield:	<b>N/A (N/A)</b>

## 22.2 Appendix II: Yahoo! Financial Overview

Yahoo! Inc.			
Consolidated Statements of Income			
	Years Ended December 31,		
	2006	2007	2008
(In thousands, except per share amounts)			
Revenues .....	\$6,425,679	\$6,969,274	\$7,208,502
Cost of revenues .....	2,675,723	2,838,758	3,023,362
Gross profit .....	3,749,956	4,130,516	4,185,140
Operating expenses:			
Sales and marketing .....	1,322,259	1,610,357	1,563,313
Product development .....	833,147	1,084,238	1,221,787
General and administrative .....	528,798	633,431	705,136
Amortization of intangibles .....	124,786	107,077	87,550
Restructuring charges, net .....	—	—	106,854
Goodwill impairment charge .....	—	—	487,537
Total operating expenses .....	2,808,990	3,435,103	4,172,177
Income from operations .....	940,966	695,413	12,963
Other income, net .....	157,034	154,011	82,838
Income before provision for income taxes, earnings in equity interests, and minority interests .....	1,098,000	849,424	95,801
Provision for income taxes .....	(458,011)	(337,263)	(262,717)
Earnings in equity interests .....	112,114	150,689	596,979
Minority interests in operations of consolidated subsidiaries .....	(712)	(2,850)	(5,765)
Net income .....	\$ 751,391	\$ 660,000	\$ 424,298
Net income per share—basic .....	\$ 0.54	\$ 0.49	\$ 0.31
Net income per share—diluted .....	\$ 0.52	\$ 0.47	\$ 0.29
Shares used in per share calculation—basic .....	1,388,741	1,338,987	1,369,476
Shares used in per share calculation—diluted .....	1,457,686	1,405,486	1,400,101
Stock-based compensation expense by function:			
Cost of revenues .....	\$ 6,621	\$ 10,628	\$ 13,813
Sales and marketing .....	155,084	246,472	182,826
Product development .....	144,807	218,207	178,091
General and administrative .....	118,418	97,120	63,113
Restructuring expense reversals .....	—	—	(30,236)
Total stock-based compensation expense .....	\$ 424,930	\$ 572,427	\$ 407,607

### Yahoo! Inc. (NasdaqGS: YHOO)

Last Trade:	<b>15.71</b>	Day's Range:	N/A - N/A
Trade Time:	<b>Jul 15</b>	52wk Range:	<b>8.94 - 23.49</b>
Change:	<b>0.00 (0.00%)</b>	Volume:	<b>1,000</b>
Prev Close:	<b>15.71</b>	Avg Vol (3m):	<b>21,393,500</b>
Open:	<b>N/A</b>	Market Cap:	<b>21.93B</b>
Bid:	<b>15.45 x 3500</b>	P/E (ttm):	<b>1,428.18</b>
Ask:	<b>15.50 x 500</b>	EPS (ttm):	<b>0.01</b>
1y Target Est:	<b>17.02</b>	Div & Yield:	<b>N/A (N/A)</b>



### 22.3 Appendix III: Market share search engines

(Comscore 2007)

Worldwide Search Top 10 December 2007 Total World Age 15+, Home and Work Locations* Source: comScore qSearch 2.0		
	Searches (MM)	Share of Searches
Total Internet	66,221	100.0
Google Sites	41,345	62.4
Yahoo! Sites	8,505	12.8
Baidu.com Inc.	3,428	5.2
Microsoft Sites	1,940	2.9
NHN Corporation	1,572	2.4
eBay	1,428	2.2
Time Warner Network	1,062	1.6
Ask Network	728	1.1
Yandex	566	0.9
Alibaba.com Corporation	531	0.8

## 22.4 Appendix IV: Profit calculations - First-price ad auction

Position	Advertiser	Cliks	Value
1	$a_1$	100	2,00
2	$a_2$	90	1,95
3	$a_3$		0,20

### Profit Postion 1 $a_2$

Bid $a_2$	Cliks	Value $a_2$	Profit
0,21	100	1,95	174
0,23	100	1,95	172
0,25	100	1,95	170
0,27	100	1,95	168
0,29	100	1,95	166
0,31	100	1,95	164
0,33	100	1,95	162
0,35	100	1,95	160
0,37	100	1,95	158
0,38	100	1,95	157
0,39	100	1,95	156
0,41	100	1,95	154
0,43	100	1,95	152
0,45	100	1,95	150
0,47	100	1,95	148
0,49	100	1,95	146
0,51	100	1,95	144
0,53	100	1,95	142
0,55	100	1,95	140
0,57	100	1,95	138
0,59	100	1,95	136
0,61	100	1,95	134
0,63	100	1,95	132
0,65	100	1,95	130
0,67	100	1,95	128
0,69	100	1,95	126
0,71	100	1,95	124
0,73	100	1,95	122
0,75	100	1,95	120
0,77	100	1,95	118

### Profit Postion 2 $a_2$

Bid $a_2$	Cliks	Value $a_2$	Profit
0,21	90	1,95	156,6

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## 24. Glossary

**Ad Auction:** Abbreviation for advertisement auction. The predominant mechanism search engines use to sell advertisements online.

**Banner ads:** A form of advertising on the World Wide Web. The advertisement is constructed from an image which is embedded into a webpage.

**Conversion Rate:** The percentage of visitors to a website who are converted into buyers. It is calculated as the number of visitors who click through divided by the actual number of conversions.

**CPA (cost-per-acquisition):** A payment model where the advertiser only pays for the amount of users who complete a transaction, such as a purchase or sign-up. This is also known as PPA.

**CPC (cost-per-click):** Cost paid by an advertiser each time a user clicks on an advertisement. This is also known as PPC.

**CPM (cost-per-mille):** A payment model where advertisers pay based on the exposure of their advertisement. Per-mille means thousands, which means that the advertiser pays per thousands impressions.

**Contextual Ads:** Ads that is related on basis of the content on a webpage instead of be directly related keywords in a search query. Many website show ads related to their content, this is contextual ads.

**CTR (click-through-rate):** The percentage of people who click on the ad out of a total number who sees it. If 100 people see you ad and 10 click on it, your CTR is 10%.

**Deep web:** The part of the world wide web which is not indexed by search engines. See indexed web.

**Exposure:** The number of times the user has the opportunity to see an advertisement, whether or not he actually sees it.

**Hits:** the number of times a webpage is viewed

**Impressions:** The number of views an ad receives.

**Landing Page** is the webpage of the advertiser that appears when a potential customer clicks on an advertisement.

**Organic listings:** Regular search results that appear in a search engine when a user types a particular keyword or phrase.

**Page Inventory:** The available ad positions on a search result page

**PPA (pay-per-acquisition):** see CPA.

**PPC (pay-per-click):** see CPC.

**Pay-per-click (PPC) Advertising:** A marketing method where a business pays a certain amount of money each time someone clicks on one of their ads displayed by a search engine or on a webpage.

**SEM (Search Engine Marketing):** Activities designed to increase the ranking of a website in search engines. Such activities include PPC advertising and regular search engine optimization.

**SEO (Search Engine Optimization):** Different techniques whereby you change the content, keywords, meta tags, etc. in order to enhance your ranking in a search engine.

**SERP:** Search Engine Result Page

**Surface web:** The visible web or the indexed web. It is the portion of the world wide web that is indexed by conventional search engines.

**Query:** The words you type in a search box when using a search engine.